QUADRATICS EQUATIONS 1

Concepts

INTRODUCTION

We are already familiar with the application of linear equations and system of linear equations in solving problems related to our day to day life. However, to solve some problem we require the application of second degree equation. In this chapter, We shall discuss equations in one variable in which the highest of the variable is two, known as quadratic equations.

1. QUADRATIC POLYNOMIAL

A polynomial of degree two is called a quadratic polynomial. E.g. $x^2 + 4$, $x^2 - 5x + 6$, $x^2 + \sqrt{3}x$, $\sqrt{2}x^2 + 2x - 6$.
A quadratic polynomial can have at most three terms namely, terms containing x^2, x and constant.

The general format of a quadratic polynomial in x is $ax^2 + bx + c$, where a, b, c are numbers and $a \neq 0$.

In quadratic polynomial $f(x) = ax^2 + bx + c$; a, b, c the called coefficients.

If for $x = \alpha$, where α is a real number, the value of quadratic expression becomes zero, then α is called zero of $ax^2 + bx + c$

A quadratic polynomial has at most two zeros.

Note : $f(x) = ax^2 + bx + c$ is also called quadratic expression.

2. POLYNOMIAL EQUATION

If f(x) be a polynomial, then f(x) = 0 is called a polynomial equation. Let f(x) be a quadratic polynomial, then f(x) = 0 is called quadratic equation. The general expression forming a quadratic equation is $ax^2 + bx + c = 0$ where a, b, $c \in R$ and $a \neq 0$.

2.1 QUADRATIC EQUATION

The second degree polynomial equations are commonly known as quadratic equation. *i.e.*, if P(x) is a quadratic polynomial, then P(x) = 0 is called a quadratic equation. The general form of a quadratic equation is $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$.

Case I : When $b \neq 0, c \neq 0$, then quadratic equation is of the type $ax^2 + bx + c = 0$.

Case II : When $b \neq 0, c = 0$, then quadratic equation is of the type $ax^2 + c = 0$.

Case III : When $b \neq 0, c = 0$, then quadratic equation is of the type $ax^2 + bx = 0$.

Case IV : When $b = 0, c = 0$, then quadratic equation is of the type $ax^2 = 0$.

2.2 ZEROS OF A QUADRATIC EQUATION

Zeros of a quadratic equation : Zeros of quadratic polynomial $ax^2 + bx + c$ where a, b, c are real number and $a \neq 0$ is found by solving the corresponding equation $ax^2 + bx + c = 0$ called a quadratic equation.

If the real number α and β are two zeros of the quadratic polynomial $ax^2 + bx + c$, then α and β are roots of the quadratic equation $ax^2 + bx + c = 0$.

There will be two roots for a quadratic equation and can be found by solving the equation $ax^2 + bx + c = 0$.

Roots are also called solutions of $ax^2 + bx + c = 0$.

Example 1

Find whether

(i) $x = 2$ is a zero of $x^2 – 5x + 6$.

(ii) $x = – 4, x = –1$ are zeros of $x^2 + 5x + 4$

(iii) $x = -\frac{1}{3}, x = -\frac{1}{4}$ are zeros of $6x^2 + 5x + 1$

Solution :

(i) $x = 2$

$\therefore x^2 + 5x + 6 = 2^2 – 5(2) + 6 = 4 – 10 + 6 = 0$

$\therefore x = 2$ is a zero of $x^2 – 5x + 6$.

(ii) $x = – 4$

$\therefore x^2 + 5x + 4 = (–4)^2 + 5(–4) + 4$

$= 16 – 20 + 4 = 0$

$\therefore x = –4$ is a zero of $x^2 + 5x + 4$

$x = – 1$

$\therefore x^2 + 5x + 4 = (–1)^2 + 5(–1) + 4 = 1 – 5 + 4 = 0$

$\therefore x = –1$ is zero of $x^2 + 5x + 4$

$\therefore x = –4$ and $x = –1$ are zeros of $x^2 + 5x + 4$.

(iii) $x = -\frac{1}{3}, \quad \therefore 6x^2 + 5x + 1 = 6\left(-\frac{1}{3}\right)^2 + 5\left(-\frac{1}{3}\right) + 1$

$= 6 \times \frac{1}{9} - \frac{5}{3} + 1 = \frac{5}{3} - \frac{5}{3} = 0$

$x = -\frac{1}{4}, \ \therefore 6x^2 + 5x + 1 = 6\left(-\frac{1}{4}\right)^2 + 5\left(-\frac{1}{4}\right) + 1 = \frac{2}{16} \neq 0$

$\therefore x = -\frac{1}{3}$ is a zero and $x = -\frac{1}{4}$ is not a zero of $6x^2 + 5x + 1$.

Example 2

If $x = 2$ and $x = 3$ are roots of equation $3x^2 – 2kx + 2m = 0$. Find the value of k and m.

Solution :

Since $x = 2$ and $x = 3$ are roots at the equation $3x^2 – 2kx + 2m = 0$

and $3(3)^2 – 2k(3) + 2m = 0$

$12 – 4k + 2m = 0$ and $27 – 6k + 2m = 0$

Solving these two equations, we get $k = \frac{15}{2}$ and $m = 9$

3. SOLUTION OF QUADRATIC EQUATION

The zeroe's of the quadratic polynomial $ax^2 + bx + c$ and the roots of the quadratic equation $ax^2 + bx + c = 0$ are same and called the solution of the quadratic equation. We can find the solution of quadratic equation by the following methods as explained below.

3.1 BY FACTORISATIO METHOD

Let $ax^2 + bx + c = 0$; $a \neq 0$ be a quadratic equation. Let it be expression into two linear factor $(px + q)$ and $(rx + s)$ where $p, q, r, s \in R$ such that $p \neq 0$ and $r \neq 0$, then $ax^2 + bx + c = (px + q)(rx + s) = 0$

$\Rightarrow$ $px + q = 0$ or $rx + s = 0$

$\Rightarrow$ $x = -\frac{q}{p}$ and $x = -\frac{s}{r}$

Example 3

Solve $81x^2 - 64 = 0$

Solution :

$81x^2 - 64 = 0$

$\Rightarrow$ $(9x)^2 - (8)^2 = 0$

$\Rightarrow$ $(9x + 8)(9x - 8) = 0$

$\therefore$ $x = -\frac{8}{9}$ or $x = \frac{8}{9}$

$\therefore$ $x = -\frac{8}{9}, \frac{8}{9}$ are solutions of $81x^2 - 64 = 0$

Example 4

Solve $5x^2 - 7x - 6 = 0$

Solution :

$5x^2 - 7x - 6 = 0$

$\Rightarrow$ $5x^2 - 10x + 3x - 6 = 0$

$\Rightarrow$ $5x(x - 2) + 3(x - 2) = 0$

$\Rightarrow$ $(x - 2)(5x + 3) = 0$

$\Rightarrow$ $x - 2 = 0$ or $5x + 3 = 0$

$\Rightarrow$ $x = 2$ or $x = -\frac{3}{5}$ $\qquad \therefore$ $x = 2, -\frac{3}{5}$ are solution of $5x^2 - 7x - 6 = 0$

Example 5

Solve $\frac{x+1}{x-1}+\frac{x-2}{x+2}=3$ $(x \neq 1, -2)$

Solution :

$$\frac{x+1}{x-1}+\frac{x-2}{x+2}=3 \quad \Rightarrow \quad \frac{x+1}{x-1}+\frac{x-2}{x+2}-3=0$$

$$\Rightarrow \frac{(x+1)(x+2)+(x-1)(x-2)-3(x-1)(x+2)}{(x-1)(x+2)}=0$$

$\Rightarrow$ $(x + 1)(x + 2) + (x - 1)(x - 2) - 3(x - 1)(x + 2) = 0$

$\Rightarrow$ $-x^2 - 3x + 10 = 0$ $\qquad \Rightarrow$ $-(x^2 + 3x - 10) = 0$

$\Rightarrow$ $x^2 + 3x - 10 = 0$

$\Rightarrow$ $(x + 5)(x - 2) = 0$ $\qquad \Rightarrow$ $x = -5$, or $x = 2$

$\therefore$ $x = -5, 2$ are the solutions of the given equation.

3.2 BY COMPLETING THE SQUARE METHOD

In this method, we rewrite a quadratic equation in the form $(x + \alpha)^2 = c^2$. This method is called the method of completing the perfect square, where c is a constant term. Following is a method to obtain the roots of the equation by using method of completing squares.

Let the quadratic equation : $ax^2 + bx + c = 0$

Dividing throughtout by a $(a \neq 0)$, we get $x^2+\frac{b}{a}x+\frac{c}{a}=0$

By adding and subtracting the square of $\left(\frac{1}{2}\text{coefficient of } x\right)^2$.

We get,

$$x^2+2.\left(\frac{b}{2a}\right)x+\left(\frac{b}{2a}\right)^2-\left(\frac{b}{2a}\right)^2+\frac{c}{a}=0$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

If $b^2 - 4ac \geq 0$, then $\sqrt{b^2-4ac}$ is a real number

$$\therefore \left(x+\frac{b}{2a}\right)=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$\therefore\ x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (This is also called quadratic formula)

Note : Here $D = b^2 - 4ac$ is called discriminant of the quadratic equation $ax^2 + bx + c = 0$

Example 6

Find the roots of quadratic equation $4x^2 + 4\sqrt{3}x + 3 = 0$ by the method of completing of square.

Solution :

We have, $4x^2 + 4\sqrt{3}x + 3 = 0$

Divide both sides by '4', we get $x^2 + \sqrt{3}x + \dfrac{3}{4} = 0$

$$\Rightarrow \quad x^2 + \sqrt{3}x = -\frac{3}{4}$$

Adding $\left(\dfrac{1}{2}\text{coefficient of } x\right)^2$ *i.e.* $\left(\dfrac{\sqrt{3}}{2}\right)^2$ on both sides

$$\Rightarrow \quad x^2 + \sqrt{3}x + \left(\frac{\sqrt{3}}{2}\right)^2 = -\frac{3}{4} + \left(\frac{\sqrt{3}}{2}\right)^2$$

$$\Rightarrow \quad \left(x + \frac{\sqrt{3}}{2}\right)^2 = \pm 0$$

$$\Rightarrow \quad x = -\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$$

Hence , roots of equation are $-\dfrac{\sqrt{3}}{2}$ and $-\dfrac{\sqrt{3}}{2}$.

Example 7

Find the roots of $a^2x^2 - 3abx + 2b^2 = 0$ by method of completing square.

Solution :

We have, $a^2x^2 - 3abx + 2b^2 = 0$

$$\Rightarrow \quad x^2 - 3\frac{b}{a}x + 2\frac{b^2}{a^2} = 0 \qquad \text{[Divide both side by } a^2\text{]}$$

$$\Rightarrow \quad x^2 - 3\frac{b}{a}x + \left(\frac{3b}{2a}\right)^2 = -\frac{2b^2}{a^2} + \left(\frac{3b}{2a}\right)^2$$

$$\Rightarrow \quad \left(x - \frac{3b}{2b}\right)^2 = -\frac{2b^2}{a^2} + \frac{9b^2}{4a^2}$$

$$\Rightarrow \left(x-\frac{3b}{2b}\right)^2=\frac{b^2}{4a^2} \qquad \Rightarrow \qquad x-\frac{3b}{2a}=\pm\frac{b}{2a}$$

$$\Rightarrow \quad x-\frac{3b}{2a}=\frac{b}{2a} \text{ or } x-\frac{3b}{2a}=-\frac{b}{2a}$$

$$\Rightarrow \quad x=\frac{3b}{2a}+\frac{b}{2a} \text{ or } x=\frac{3b}{2a}-\frac{b}{2a}$$

$$\Rightarrow \quad x=\frac{2b}{a} \text{ or } x=\frac{b}{a}$$

Example 8

Solve $abx^2+(b^2-ac)x-bc=0$ by using quadratic formula.

Solution :

We have, $abx^2+(b^2-ac)x-bc=0$

Comparing it with $Ax^2+Bx+C=0$, we have

$A = ab$, $B = b^2 - ac$, $C = -bc$

$D = B^2 - 4AC = (b^2 - ac)^2 - 4(ab)(-bc)$

$= b^4 + a^2c^2 - 2ab^2c + 4ab^2c \qquad \Rightarrow \qquad (b^2 + ac)^2 > 0$

So, the roots of the given equation are real and given by

$$\alpha=\frac{-B+\sqrt{D}}{2A}=\frac{-(b^2-ac)+\sqrt{(b^2+ac)^2}}{2ab}$$

$$\frac{-b^2+ac+b^2+ac}{2ab}=\frac{2ac}{2ab}=\frac{c}{d}$$

$$\beta=\frac{-B-\sqrt{D}}{2A}=\frac{-b^2+ac-b^2-ac}{2ab}=\frac{-2b^2}{2ab}=-\frac{b}{a}$$

$$\therefore \quad x=\frac{c}{b},\frac{-b}{a}$$

Example 9

The sum of squares of two consecutive positive integers is 221. Find the integers.

Solution :

Let x be one of the positive integers. Then the other is $x + 1$

$\therefore$ Sum of squares of the integers $= x^2 + (x + 1)^2 = 221$

$\therefore$ $x^2 + x^2 + 2x + 1 - 221 = 0$

$2x^2 + 2x - 220 = 0$

$2(x^2 + x - 110) = 0$

$\therefore$ $(x - 10)(x + 11) = 0$ $\therefore$ $x = 10$ or $x = -11$

$\therefore$ consecutive positive integers are 10 and –11.

Example 10

A takes 6 days less than the time taken by B to finish a piece of work. If both A and B together can finish it in 4 days, find the time taken by B to finish the work.

Solution :

Let B takes x days to complete the work. Then A takes (x – 6) days to do the same work.

$\Rightarrow \dfrac{1}{x-6} + \dfrac{1}{x} = \dfrac{1}{4}$

$\Rightarrow \dfrac{x+x-6}{(x-6)x} = \dfrac{1}{4}$ $\Rightarrow \dfrac{2x-6}{x^2-6x} = \dfrac{1}{4}$

$x^2 - 6x = 8x - 24$ $\Rightarrow$ $x^2 - 14x + 24 = 0$

$\Rightarrow$ $x^2 - 12x - 2x + 24 = 0$

$\Rightarrow$ $(x - 2)(x - 12) = 0$

$\Rightarrow$ $x = 2$ or $x = 12$

But x can't be 2, so x = 2 is rejected take x = 12

Hence, B alone can finish the work in 12 days.

4. NATURE OF ROOTS

We have already learnt that the roots of a quadratic equation with real coefficients can be real or complex. When the roots are real, they can be rational or irrational and, also, they can be equal or unequal.

Let α, β be the two roots of the quadratic equation $ax^2 + bx + c = 0$ and we have the quadratic formula to find α, β, *i. e.*, $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a \neq 0$ and $b^2 - 4ac > 0$

since $b^2 - 4ac$ determines nature of roots of equation $ax^2 + bx + c = 0$, hence it called discriminant of the quadratic equation denoted by D.

So, a quadratic equation $ax^2 + bx + c = 0$ has

(i) complex conjugates roots, when $D < 0$, means no real roots.

(ii) rational and equal roots, when $D = 0$

(iii) rational and unequal roots, when $D > 0$ and a perfect square

(iv) irrational and uneuqal roots, when $D > 0$ and not a perfect square

Example 11

Find the nature of roots of following equations :

(i) $x^2 + 3x + 2$ (ii) $x^2 + 5x + 7$

Solution :

(i) Here, $D = b^2 - 4ac = 9 - (4 \times 1 \times 2) = 1$, Hence $D > 0$ so roots are real

(ii) Here, $D = b^2 - 4ac = 25 - (4 \times 1 \times 7) = -3$, Hence $D < 0$ so roots of the equation are not real, means roots will be complex conjugates

Example 12

Find the values of p for which the quadratic equation $6x^2 + px + 6 = 0$ has real roots.

Solution :

$D = b^2 - 4ac = p^2 - 4 \times 6 \times 6 = p^2 - 144$

As the equation has real roots $D \geq 0$

$\therefore\ p^2 - 12^2 \geq 0 \qquad \Rightarrow\ (p + 12)(p - 12) \geq 0 \qquad .....(1)$

(1) holds good if

(i) $p + 12 \geq 0$ and $p - 12 \geq 0$

$\therefore\ p \geq -12,\ p \geq 12$

$\therefore\ p \geq 12$ or $P \geq -12$

(ii) $p + 12 \leq 0$ and $p - 12 \leq 0$

$\therefore\ p \leq -12$ and $p \leq 12 \qquad \therefore\ p \leq -12$

$\therefore$ Required values of p are $p \leq -12$ or $p \geq 12$

Example 13

Determine whether the following quadratic equation have real roots and if they have, find them.

(i) $2x^2 + 11x - 6 = 0$

(ii) $x^2 - 6x + 9 = 0$

(iii) $x^2 + x + 1 = 0$

(iv) $x^2 - 4x - 9 = 0$

Solution :

(i) $2x^2 + 11x - 6 = 0$

Comparing this equation with $ax^2 + bx + c = 0$

We have $a = 2,\ b = 11,\ c = -6$

$\therefore$ Discriminant $D = b^2 - 4ac = 11^2 - 4(2)(-6) = 121 + 48 = 169$

$\therefore$ $D > 0$, given equation will have two distinct real roots say α, β

given by $\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a} = \frac{-11+\sqrt{169}}{4} = \frac{-11+13}{4} = \frac{1}{2}$

$\beta = \frac{-b-\sqrt{b^2-4ac}}{2a} = \frac{-11-\sqrt{169}}{4} = \frac{-11-13}{4} = -6$

$\therefore$ the two roots are $\frac{1}{2}$ and –6

(ii) $x^2 - 6x + 9 = 0$

$\therefore$ $a = 1, b = -6, c = 9$

$\therefore$ $D = b^2 - 4ac = (-6)^2 - 4(1)(9) = 36 - 36 = 0$

$\therefore$ Equation has a repeated root given by $\alpha = -\frac{b}{2a} = -\frac{(-6)}{2\times 1} = 3$

(iii) $x^2 + x + 1 = 0$

$a = 1, b = 1, c = 1$

$\therefore$ $D = b^2 - 4ac = 1^2 - 4(1)(1) = -3 < 0$

$\therefore$ The equation does not have real roots.

(iv) $x^2 - 4x - 9 = 0$

$a = 1, b = -4, c = -9$

$D = b^2 - 4ac = (-4)^2 - 4(1)(-9) = 16 + 36 = 52 > 0$

$\therefore$ The equation has two roots given by

$\frac{4+\sqrt{52}}{2}, \frac{4-\sqrt{52}}{2}$ $\quad\therefore\quad x = \frac{4+\sqrt{52}}{2}, \frac{4-\sqrt{52}}{2}$

$\frac{4+2\sqrt{13}}{2}, \frac{4-2\sqrt{13}}{2}$ *i.e.* $2+\sqrt{13}, 2-\sqrt{13}$ are the required solutions.

5. RELATION BETWEEN ROOTS AND COEFFICIENTS OF A QUADRATIC EQUAITON

If α, β are roots fo $ax^2 + bx + c = 0$ then

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}, \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$\therefore$ $\alpha + \beta$ = sum of roots $= -\frac{b}{a} = -\frac{(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$

$\therefore$ $\alpha\beta$ = product of roots $= \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

Focus Point

(i) A quadratic equation whose roots are α and β is given by $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

i.e. x^2 – (sum of roots)x + product of roots = 0

(ii) Some important formulae :

1. $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
2. $(\alpha^3 + \beta^3) = (\alpha + \beta)^3 - 3\alpha\beta\,(\alpha + \beta)$
3. $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$
4. $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$
5. $\alpha^2 - \beta^2 = (\alpha - \beta)(\alpha + \beta) = (\alpha + \beta)\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$
6. $\alpha^3 - \beta^3 = (\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2)$
7. $\alpha^4 - \beta^4 = (\alpha + \beta)(\alpha - \beta)(\alpha^2 + \beta^2) = (\alpha + \beta)(\alpha - \beta)[(\alpha + \beta)^2 - 2\alpha\beta]$
8. $\alpha^5 + \beta^5 = (\alpha^3 + \beta^3)(\alpha^2 + \beta^2) - \alpha^2\beta^2(\alpha + \beta)$

Example 14

If α, β are roots of $ax^2 + bx + c = 0$. Find the quadratic equation whose roots are.

(i) $2\alpha,\ 2\beta$ (ii) $\alpha + 3,\ \beta + 3$ (iii) $\frac{\alpha}{4}, \frac{\beta}{4}$ (iv) $\frac{1}{\alpha}, \frac{1}{\beta}$

Solution :

α, β are roots of $ax^2 + bx + c = 0$

$$\alpha + \beta = -\frac{b}{a},\ \ \alpha\beta = \frac{c}{a}$$

(i) We have to find the equation whose roots are $2\alpha, 2\beta$

$\therefore$ for the required equation

$$\text{sum of roots} = 2\alpha + 2\beta = 2(\alpha + \beta) = 2\left(-\frac{b}{a}\right) = -\frac{2b}{a}$$

$$\text{product of roots} = (2\alpha)(2\beta) = 4\alpha\beta = 4\left(\frac{c}{a}\right) = \frac{4c}{a}$$

$\therefore$ The equation whose roots are $2\alpha, 2\beta$ is.

x^2 – (sum of roots)x + (product of roots) = 0

$\therefore\ x^2 + \frac{2b}{a}x + \frac{4c}{a} = 0$ *i.e.* $ax^2 + 2bx + 4c = 0$

(ii) Sum $= \alpha + 3 + \beta + 3 = \alpha + \beta + 6 = -\frac{b}{a} + 6 = \frac{-b+6a}{a}$

product $= (\alpha + 3)(\beta + 3) = \alpha\beta + 3\alpha + 3\beta + 9$

$= \alpha\beta + 3(\alpha + \beta) + 9 = \frac{c}{a} - \frac{3b}{a} + 9 = \frac{c-3b+9a}{a}$

$\therefore$ required equation is $x^2 -$ (sum of roots)$x +$ product of roots $= 0$

$$x^2 - \left(\frac{-b+6a}{a}\right)x + \left(\frac{c-3b+9a}{a}\right) = 0$$

i.e. $ax^2 - (-b + 6a)x + (c - 3b + 9a) = 0$

(iii) sum $= \frac{\alpha}{4} + \frac{\beta}{4} = \frac{\alpha+\beta}{4} = \frac{-b/a}{4} = \frac{-b}{4a}$

Product $= \frac{\alpha}{4} \times \frac{\beta}{4} = \frac{c}{16a}$

$\therefore$ Required equation is $x^2 - \left(-\frac{b}{4a}\right)x + \frac{c}{16a} = 0$ *i.e.* $16ax^2 + 4bx + c = 0$

(iv) sum $= \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-b/a}{c/a} = -\frac{b}{c}$

product $= \frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{c/a} = \frac{a}{c}$

$\therefore$ required equation is $x^2 - \left(-\frac{b}{c}\right)x + \frac{a}{c} = 0$

i.e. $cx^2 + bx + a = 0$

Example 15

If sum of the roots of the equation $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$ is zero, then prove that the product of roots is $-\frac{1}{2}(a^2 + b^2)$

Solution :

We have $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$

$\Rightarrow \quad x^2 + (a + b - 2c)x + (ab - bc - ca) = 0$

Let α, β be the roots of the equation.

Given $\alpha + \beta = 0 \quad \Rightarrow -(a + b - 2c) = 0 \quad \Rightarrow c = \frac{a+b}{2}$ .....(i)

$\alpha\beta = ab - bc - ca = ab - c(a+b) = ab - \frac{(a+b)^2}{2}$ Using equation (i)

$= \frac{2ab - (a+b)^2}{2} = -\frac{1}{2}(a^2 + b^2)$

Example 16

For the quadratic equation $ax^2 + bx + x = 0$; $a \neq 0$ find the condition that :

(i) one root is reciprocal of the other

(ii) one root is n times the other root.

Solution :

We have $ax^2 + bx + c = 0$; $a \neq 0$

Let α, β be root of the equation then $\alpha + \beta = -\frac{b}{a}$ ......(i)

and $\alpha\beta = \frac{c}{a}$ ......(ii)

(i) Let $\beta = \frac{1}{\alpha}$, then $\alpha\beta = 1$

$\Rightarrow \frac{c}{a} = 1$ or $c = a$, which is required condition

(ii) Let $\beta = n\alpha$

Now from equation (i) and (ii)

$\alpha + n\alpha = -\frac{b}{a}$ and $\alpha . n\alpha = \frac{c}{a}$

$\alpha = \frac{-b}{a(n+1)}$ and $n\alpha^2 = \frac{c}{a}$

$\Rightarrow n\left[\frac{-b}{a(n+1)}\right]^2 = \frac{c}{a}$ $\Rightarrow \frac{nb^2}{a^2(n+1)^2} = \frac{c}{a}$

$\Rightarrow ac(n+1)^2 = b^2n$

Which is the required condition

6. FORMATION OF A QUADRATIC EQUATION FROM GIVEN ROOTS

If α, β be the roots of a quadratic equation then the quadratic equation is given by:

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$ *i.e.*

$x^2 -$ (sum of roots)x + product of roots $= 0$

SE. 1

Deepak and sudhir together have 26 marbles. Hoth of them lost 3 marbles each, and the product of the number of marbles they now have is 91. We want to find out how many marbles they had to start with. Represent the above problem mathematically in terms of a quadratic equation.

Ans. Let the number of marbles Deepak had = x, Then the number of marbles Sudhir had in the beginning = $(26 - x)$.

[$\because$ Deepak and Sudhir together have 26 marbles]

The number of marbles left with Deepak after losing 3 marbles = $(x - 3)$

The number of marbles left with Sudhir after losing 3 marbles = $(26 - x - 3) = (23 - x)$

According to question, we have $(x - 3)(23 - x) = 91$

$\Rightarrow \quad 23x - x^2 - 69 + 3x = 91$

$\Rightarrow \quad -x^2 + 26x - 69 - 91 = 0$

$\Rightarrow \quad -x^2 + 26x - 160 = 0$

$\Rightarrow \quad x^2 - 26x + 160 = 0$

Hence, the required quadratic equation is

$x^2 - 26x + 160 = 0$

SE. 2

A train travels a distance of 720 km at a uniforms speed. If the speed has been 12 km/hr less, then it would have taken 2 hrs more to cover the same distance. Represent the above problem mathematically in terms of a quadratic equation.

Ans. Let the uniform speed of the train be x km/hr.

Time taken to travel $720\,\text{km} = \dfrac{720}{x}$ hrs

When the speed is reduced by 12 km/hr, the time taken to travel $720\,\text{km} = \dfrac{720}{(x-12)}$ hrs

According to question, $\dfrac{720}{(x-12)} - \dfrac{720}{x} = 3$

$\Rightarrow \quad 720 \times \left\{\dfrac{1}{(x-12)} - \dfrac{1}{x}\right\} = 3$

$\Rightarrow \quad 720 \times \left\{\dfrac{x-(x-12)}{x(x-12)}\right\} = 3$

$\Rightarrow \quad 720 \times \dfrac{12}{x(x-12)} = 3$

$\Rightarrow \quad 720 \times 12 = 3 \times x\,(x - 12)$

$\Rightarrow \quad x(x - 12) = \dfrac{720 \times 12}{3}$

$\Rightarrow \quad x^2 - 12x = 720 \times 4$

$\Rightarrow \quad x^2 - 12x - 2880 = 0$ is the required quadratic equation

SE. 3

If roots of the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal, prove that $\dfrac{a}{b} = \dfrac{c}{d}$.

Ans. $(a^2 + b^2)x^2 - 2(ac + bd)x + c^2 + d^2 = 0$

Here $A = a^2 + b^2$, $B = -2(ac + bd)$, $C = c^2 + d^2$

$\because$ Roots are equal $\quad \therefore D = 0 \Rightarrow B^2 - 4AC = 0$

$\Rightarrow \quad 4(ac + bd)^2 - 4(a^2 + b^2)(c^2 + d^2) = 0$

$\Rightarrow \quad 4[a^2c^2 + b^2d^2 + 2abcd - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2] = 0$

$\Rightarrow \quad -4[a^2d^2 + b^2c^2 - 2abcd] = 0$

$\Rightarrow \quad (ad - bc)^2 = 0 \Rightarrow ad = bc \Rightarrow \dfrac{a}{b} = \dfrac{c}{d}$

SE. 4

Solve for x : $4x^2 - 4a^2x + (a^4 - b^4) = 0$.

Ans. We have, $4x^2 - 4a^2x + (a^4 - b^4) = 0$.

$\Rightarrow \quad (2x)^2 - 2(2x)(a^2) + (a^2)^2 - b^4 = 0$

$\Rightarrow \quad (2x - a^2)^2 - (b^2)^2 = 0$

$\Rightarrow \quad (2x - a^2 + b^2)(2x - a^2 - b^2) = 0$

$[a^2 - b^2 = (a + b)(a - b)]$

If $2x - a^2 + b^2 = 0$, then $x = \frac{a^2 - b^2}{2}$

and if $2x - a^2 - b^2 = 0$, then $x = \frac{a^2 + b^2}{2}$

Hence, $x = \frac{a^2 - b^2}{2}, \frac{a^2 + b^2}{2}$

SE. 5

Solve : $5^{(x+1)} + 5^{(2-x)} = 5^3 + 1$

Ans. We have, $5^{(x+1)} + 5^{(2-x)} = 5^3 + 1$

$\Rightarrow \quad 5^x . 5 + 5^2 . 5^{-x} = 126 \quad \Rightarrow 5^x . 5 + \frac{25}{5^x} = 126$

Let $5^x = y$

$\Rightarrow \quad 5y + \frac{25}{y} = 126$

$\Rightarrow \quad 5y^2 - 126y + 25 = 0$

$\Rightarrow \quad 5y^2 - 125y - y + 25 = 0$

$\Rightarrow \quad 5y(y - 25) - 1(y - 25) = 0$

$\Rightarrow \quad (y - 25)(5y - 1) = 0$

If $y - 25 = 0$, then $y = 25$, *i. e.* $5^x = 25$

or $5^x = 5^2 \quad \therefore x = 2$

If $5y - 1 = 0$, then $y = \frac{1}{5}$, *i. e.* $5^x = 5^{-1}$

$\therefore x = -1$

Hence, $x = 2, -1$

SE. 6

Solve for

$x : \frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} + \frac{1}{(x-3)(x-4)} = \frac{1}{6}$

Ans. we have,

$\left[\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} + \frac{1}{(x-3)(x-4)} = \frac{1}{6}\right]$

$\Rightarrow \quad \frac{x-3+x-1}{(x-1)(x-2)(x-3)} + \frac{1}{(x-3)(x-4)} = \frac{1}{6}$

$\Rightarrow \quad \frac{2x-4}{(x-1)(x-2)(x-3)} + \frac{1}{(x-3)(x-4)} = \frac{1}{6}$

$\Rightarrow \quad \frac{2}{(x-1)(x-3)} + \frac{1}{(x-3)(x-4)} = \frac{1}{6}$

$\Rightarrow \quad \frac{2x-8+x-1}{(x-1)(x-3)(x-4)} = \frac{1}{6}$

$\Rightarrow \quad \frac{3(x-3)}{(x-1)(x-3)(x-4)} = \frac{1}{6}$

$\Rightarrow \quad \frac{3}{(x-1)(x-4)} = \frac{1}{6}$

$\Rightarrow \quad (x-1)(x-4) = 18 \quad \Rightarrow \quad x^2 - 5x + 4 = 18$

$\Rightarrow \quad x^2 - 5x - 14 = 0$

Using middle term splitting we get

$x^2 - 7x + 2x - 14 = 0$

$\Rightarrow \quad x(x-7) + 2(x-7) = 0$

$\Rightarrow \quad (x-7)(x+2) = 0$

If $x - 7 = 0$, then $x = 7$ and if $x + 2 = 0$, then $x = -2$

Hence, $x = 7, -2$.

SE. 7

Solve for x, using quadratic formula.

(i) $abx^2 + (b^2 - ac)x - bc = 0$

(ii) $9x^2 - 9(a+b)x + (2a^2 + 5ab + 2b^2) = 0$

Ans. (i) $abx^2 + (b^2 - ac)x - bc = 0$

Here, $A = ab$, $B = b^2 - ac$, $C = -bc$

$\therefore D = B^2 - 4AC = (b^2 - ac)^2 - 4(ab)(-bc)$

$= b^4 - 2b^2ac + a^2c^2 + 4ab^2c$

$= b^4 + 2b^2ac + a^2c^2 = (b^2 + ac)^2$

$$x = \frac{-B \pm \sqrt{D}}{2A} = \frac{-(b^2 - ac) \pm \sqrt{(b^2 + ac)^2}}{2ab}$$

$\Rightarrow \quad x = \frac{-(b^2 - ac) + (b^2 + ac)}{2ab}$ or $\frac{-(b^2 - ac) - (b^2 + ac)}{2ab}$

$\Rightarrow \quad x = \frac{2ac}{2ab}$ or $x = \frac{-2b^2}{2ab} \quad \therefore x = \frac{c}{b}$ or $\frac{-b}{a}$

(ii) $9x^2 - 9(a+b)x + (2a^2 + 5ab + 2b^2) = 0$

Here $A = 9$, $B = -9(a+b)$, $C = 2a^2 + 5ab + 2b^2$

$\therefore \quad D = B^2 - 4AC = [-9(a+b)]^2 - 4.9(2a^2 + 5ab + 2b^2)$

$= 9[9a^2 + 18ab + 9b^2 - 8a^2 - 20ab - 8b^2]$

$= 9[a^2 - 2ab + b^2] = [3(a-b)]^2$

$\therefore$ Root are given by

$$x=\frac{-B\pm\sqrt{D}}{2A}=\frac{9(a+b)\pm3(a-b)}{18}$$

$$\Rightarrow\quad x=\frac{9(a+b)+3(a-b)}{18}\quad\text{or}$$

$$x=\frac{9(a+b)-3(a-b)}{18}$$

$$\Rightarrow\quad x=\frac{12a+6b}{18}\quad\text{or}\quad x=\frac{6a+12b}{18}$$

$$\therefore\quad x=\frac{2a+b}{3}\quad\text{or}\quad x=\frac{a+2b}{3}$$

SE. 8

Find roots of following equations.

(i) $\frac{x-1}{x-2}+\frac{x-3}{x-4}=3\frac{1}{3};(x\neq2,4)$

(ii) $\frac{1}{x-3}-\frac{1}{x+5}=\frac{1}{6};(x\neq3,-5)$

Ans. (i) $\frac{x-1}{x-2}+\frac{x-3}{x-4}=3\frac{1}{3};(x\neq3,-5)$

$$\Rightarrow\quad\frac{(x-1)(x-4)+(x-2)(x-3)}{(x-2)(x-4)}=\frac{10}{3}$$

$$\Rightarrow\quad\frac{x^2-5x+4+x^2-5x+6}{x^2-6x+8}=\frac{10}{3}$$

$$\Rightarrow\quad\frac{2(x^2-5x+5)}{x^2-6x+8}=\frac{10}{3}$$

$$\Rightarrow\quad\frac{x^2-5x+5}{x^2-6x+8}=\frac{5}{3}$$

$\Rightarrow\quad 5x^2-30x+40=3x^2-15x+15$

$\Rightarrow\quad 2x^2-15x+25=0$

$\Rightarrow\quad 2x^2-10x-5x+25=0$

$\Rightarrow\quad 2x(x-5)-5(x-5)=0$

$\Rightarrow\quad (x-5)(2x-5)=0$

$\Rightarrow\quad x-5=0\quad\text{or}\quad 2x-5$

$\Rightarrow\quad x=5\quad\text{or}\quad x=\frac{5}{2}$

$\therefore$ Required roots are $\frac{5}{2}$ and 5.

(ii) $\frac{1}{x-3}-\frac{1}{x+5}=\frac{1}{6};(x\neq3,-5)$

$$\Rightarrow\quad\frac{8}{x^2+2x-15}=\frac{1}{6}$$

$$\Rightarrow\quad\frac{(x+5)-(x-3)}{(x-3)(x+5)}=\frac{1}{6}$$

$$\Rightarrow\quad\frac{8}{x^2+2x-15}=\frac{1}{6}$$

$\Rightarrow\quad x^2+2x-15=48$

$\Rightarrow\quad x^2+2x-63=0$

$\Rightarrow\quad x^2+9x-7x-63=0$

$\Rightarrow\quad x(x+9)-7(x+9)=0$

$\Rightarrow\quad (x+9)(x-7)=0$

$\Rightarrow\quad x+9=0\quad\text{or}\quad x-7=0$

$\Rightarrow\quad x=-9\quad\text{or}\quad x=7$

$\therefore$ Roots of given equation are –9 and 7

SE. 9

If – 4 is a root of the quadratic equation $x^2+px-4=0$ and the quadratic equation $x^2+px+k=0$ has equal roots, find the value of k.

Ans. Since – 4 is a root of the equation

$x^2+px-4=0$

$\therefore\ (-4)^2+p\times(-4)-4=0$

[$\because$ A root always satisfies the equation]

$\Rightarrow\quad 16-4p-4=0$

$\Rightarrow\quad 4p=12$

$\Rightarrow\quad p=3$

The equation $x^2+px+k=0$ has equal roots. Here $a=1$, $b=p$ and $c=k$

$\therefore$ Discriminant $=0$

$\Rightarrow\quad p^2-4k=0$

$\Rightarrow\quad 9-4k=0$

$\Rightarrow\quad k=\frac{9}{4}$

SE. 10

If the sum of first n even natural numbers is 420, find the value of n.

Ans. We have 2 + 4 + 6 + 8 + ... to n terms = 420

$\Rightarrow \frac{n}{2}[2\times2+(n-1)\times2]=420$

$\Rightarrow n(2+n-1)=420$

$\Rightarrow n(n+1)=420$

$\Rightarrow n^2+n-420=0$

$\Rightarrow n^2+21n-20n-420=0$

$\Rightarrow n(n+21)(n-20)=0$

$\Rightarrow n=20,-21$

$\because$ n is a natural number

$\therefore$ $n>0$ $\therefore$ $n=20$

SE. 11

A swimming pool is filled with three pipes with uniform flow. The first two pipes operating simultaneously, fil the pool in same time during which the pool is filled by the third pipe alone. The second pipe fills pool the five hours faster than the first pipe and four hours slower than the third pipe. Find the time required by each pipe to fill the pool separately.

Ans. Let V be the volume of the pool and x be the number of hours required by the second pipe alone to fill the pool. Then, the first pipe takes (x + 5) hours, while the third pipe takes (x – 4) hours to fill the pool. So, the parts of the pool filled by the first, second and third pipes in one hour are respectively

$\frac{V}{x+5},\frac{V}{x}$ and $\frac{V}{x-4}$

Let the time taken by the first and second pipes to fill the pool simultaneously be t hours. Then, the third pipe alos takes the same time to fill the pool.

$\therefore \left(\frac{V}{x+5}+\frac{V}{x}\right)t=\text{Volume of the pool}=\frac{V}{x-4}t$

$\Rightarrow \left(\frac{V}{x+5}+\frac{V}{x}\right)t=\frac{V}{x-4}t$

$\Rightarrow \frac{1}{x+5}+\frac{1}{x}=\frac{1}{x-4}$

$\Rightarrow \frac{x+x+5}{(x+5)x}=\frac{1}{x-4}$

$\Rightarrow (2x+5)(x-4)=x^2+5x$

$\Rightarrow x^2-8x-20=0$

$\Rightarrow x^2-10x+2x-20=0$

$\Rightarrow x(x-10)+2(x-10)=0$

$\Rightarrow (x-10)(x+2)=0$

$\Rightarrow x=10$ or $x=-2$. But, x cannot be negative.

So, x = 10

Hence, the timings required by first, second and third pipes to fill the pool individually are 15 hours, 10 hours and 6 hours respectively.

EXERCISE - 4.1

NS. 1

Check whether the following are quadratic equations.

(i) $(x+1)^2 = 2(x-3)$

(ii) $x^2 - 2x = (-2)(3-x)$

(iii) $(x-2)(x+1) = (x-1)(x+3)$

(iv) $(x-3)(2x+1) = x(x+5)$

(v) $(2x-1)(x-3) = (x+5)(x-1)$

(vi) $x^2 + 3x + 1 = (x-2)^2$

(vii) $(x+2)^3 = 2x(x^2-1)$

(viii) $x^3 - 4x^2 - x + 1 = (x-2)^3$

Ans. **(i)** $x^2 + 2x + 1 = 2x - 6$

$\Rightarrow$ $x^2 + 2x + 1 - 2x + 6 = 0$

$\Rightarrow$ $x^2 + 7 = 0$

Since $x^2 + 7$ is a quadratic polynomial

$\therefore$ $(x+1)^2 = 2(x-3)$ is a quadratic equation.

(ii) $x^2 - 2x = -6 + 2x$

$\Rightarrow$ $x^2 - 4x + 6 = 0$

Since $x^2 - 4x + 6$ is a quadratic polynomial

$\therefore$ $x^2 - 2x = (-2)(3-x)$ is a quadratic equation.

(iii) $x^2 - x - 2 = x^2 + 2x - 3$

$\Rightarrow$ $x^2 - x - 2 - x^2 - 2x + 3 = 0$

$\Rightarrow$ $-3x + 1 = 0$

Since $-3x + 1$ is a linear polynomial

$\therefore$ $(x-2)(x+1) = (x-1)(x+3)$ is not a quadratic equation

(iv) $2x^2 + x - 6x - 3 = x^2 + 5x$

$\Rightarrow$ $2x^2 - 5x - 3 - x^2 - 5x = 0$

$\Rightarrow$ $x^2 - 10x - 3 = 0$

Since $x^2 - 10x - 3$ is a quadratic polynomial.

$\therefore$ $(x-3)(2x+1) = x(x+5)$ is a quadratic equation

(v) $2x^2 - 6x - x + 3 = x^2 - x + 5x - 5$

$\Rightarrow$ $2x^2 - x^2 - 6x - x + x - 5x + 3 + 5 = 0$

$\Rightarrow$ $x^2 - 11x + 8 = 0$

Since $x^2 - 11x + 8$ is a quadratic polynomial

$\therefore$ $(2x-1)(x-3) = (x+5)(x-1)$ is a quadratic equation

(vi) $x^2 + 3x + 1 = x^2 - 4x + 4$

$\Rightarrow$ $x^2 + 3x + 1 - x^2 + 4x - 4 = 0$

$\Rightarrow$ $7x - 3 = 0$

Since $7x - 3$ is a linear polynomial

$\therefore$ $x^2 + 3x + 1 = (x-2)^2$ is not a quadratic equation

(vii) $x^3 + 3x^2(2) + 3x(2)^2 + (2)^3 = 2x^3 - 2x$

$\Rightarrow$ $x^3 + 6x^2 + 12x + 8 = 2x^3 - 2x$

$\Rightarrow$ $x^3 + 6x^2 + 12x + 8 - 2x^3 + 2x = 0$

$\Rightarrow$ $-x^3 + 6x^2 + 14x + 8 = 0$

Since $-x^3 + 6x^2 + 14x + 8$ is a polynomial of degree 3

$\therefore$ $(x+2)^3 = 2x(x^2-1)$ is not a quadratic equation.

(viii) $x^3 - 4x^2 - x + 4 = x^3 + 3x^2(-2) + 3x(-2)^2 + (-2)^3$

$\Rightarrow$ $x^3 - 4x^2 - x + 1 = x^3 - 6x^2 + 12x - 8$

$\Rightarrow$ $x^3 - 4x^2 - x + 1 - x^3 + 6x^2 - 12x + 8 = 0$

$\Rightarrow$ $2x^2 - 13x + 9 = 0$

Since $2x^2 - 13x + 9$ is a quadratic polynomial

$\therefore$ $x^3 - 4x^2 - x + 1 = (x-2)^3$ is a quadratic equation.

NS. 2

Represent the following situations in the form of quadratic equations

(i) The area of a rectangular plot is 528 m^2. The length of the plot (in meters) is one more than twice its breadth. We need to find the length and breadth of the plot.

(ii) The product of two consecutive positive integers is 306. We need to find the integers.

(iii) Rohan's mother is 26 years older than him.The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/hr less, then it would have taken 3 hours more to cover the same distance. We need to find the speed the train.

Ans. **(i)** Let the breadth = x meters

$\because$ Length = 2(Breadth) + 1

$\therefore$ Length = (2x + 1) meters

Since Length × Breadth = Area

$\therefore$ $(2x + 1) \times x = 528$

$\Rightarrow$ $2x^2 + x = 528$

$\Rightarrow$ $2x^2 + x - 528 = 0$

Thus, the required quadratic equation is

$2x^2 + x - 528 = 0$

(ii) Let the two consecutive positive intergers be x and (x + 1)

$\because$ Product of the postive integers = 360

$\therefore$ $x(x + 1) = 306$

$\Rightarrow$ $x^2 + x = 306$

$\Rightarrow$ $x^2 + x - 306 = 0$

Thus, the required quadratic equation is

$x^2 + x - 306 = 0$

(iii) Let the present age of Rohan be x years

$\therefore$ Mother's age = (x + 26) years

After 3 years, Rohan's age = (x + 3) years.

Mother's age = [(x + 26) + 3] years = (x + 29) years

According to the condition, [Product of their ages after 3 years] = 360

$\Rightarrow$ $(x + 3) \times (x + 29) = 360$

$\Rightarrow$ $x^2 + 29x + 3x + 87 = 360$

$\Rightarrow$ $x^2 + 29x + 3x + 87 - 360 = 0$

$\Rightarrow$ $x^2 + 32x - 273 = 0$

Thus, the required quadratic equation is.

$x^2 + 32x - 273 = 0$

(iv) Let the speed of the train = u km/hr

Distance covered = 480 km

Time taken = Distance ÷ Speed

= (480 ÷ u) hours

In second case, Speed = (u – 8) km/hr

$\therefore$ Time taken $\dfrac{\text{Dis tan ce}}{\text{Speed}} = \dfrac{480}{(u-8)}$ hours

According to the condition, $\dfrac{480}{u-8} - \dfrac{480}{u} = 3$

$\Rightarrow$ $480u - 480(u - 8) = 3u(u - 8)$

$\Rightarrow$ $480u - 480u + 3840 = 3u^2 - 24u$

$\Rightarrow$ $3840 - 3u^2 + 24u = 0$

$\Rightarrow$ $u^2 - 8u - 1280 = 0$

Thus, the required quadratic equation is.

$u^2 - 8u - 1280 = 0$

EXERCISE - 4.2

NS. 1

Find the roots of the following quadratic equations by factorisation

(i) $x^2 - 3x - 10 = 0$

(ii) $2x^2 + x - 6 = 0$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

(iv) $2x^2 - x + \dfrac{1}{8} = 0$

(v) $100x^2 - 20x + 1 = 0$

Ans. **(i)** We have, $x^2 - 3x - 10 = 0$

$\Rightarrow$ $x^2 - 5x + 2x - 10 = 0$

$\Rightarrow$ $x(x - 5) + 2(x - 5) = 0$

$\Rightarrow$ $(x - 5)(x + 2) = 0$

Either $x - 5 = 0$

$\Rightarrow$ $x = 5$ or $x + 2 = 0$ $\Rightarrow$ $x = -2$

Thus, the required roots are x = 5 and x = –2

(ii) We have $2x^2 + x - 6 = 0$

$\Rightarrow$ $2x^2 + 4x - 3x - 6 = 0$

$\Rightarrow$ $2x(x + 2) - 3(x + 2) = 0$

$\Rightarrow$ $(x + 2)(2x - 3) = 0$

Either $x + 2 = 0$

$\Rightarrow \quad x = -2 \text{ or } 2x - 3 = 0 \quad \Rightarrow \quad x = \frac{3}{2}$

Thus, the required roots are $x = -2$ and $x = \frac{3}{2}$

(iii) We have $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow \quad \sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$

$\Rightarrow \quad \sqrt{2}x^2 + (\sqrt{2}.\sqrt{2})x + 5x + 5.\sqrt{2} = 0$

$\Rightarrow \quad \sqrt{2}x[x + \sqrt{2}] + 5[x + \sqrt{2}] = 0$

$\Rightarrow \quad (x + \sqrt{2})(\sqrt{2}x + 5) = 0$

Either $x + \sqrt{2} = 0$

$\Rightarrow \quad x = -\sqrt{2} \text{ or } \sqrt{2}x + 5 = 0$

$\Rightarrow \quad x = -\frac{5}{\sqrt{2}} \quad \Rightarrow \quad x = \frac{-5\sqrt{2}}{2}$

Thus, the required roots are $x = -\sqrt{2}$ and $x = \frac{-5\sqrt{2}}{2}$

(iv) We have, $2x^2 - x + \frac{1}{8} = 0$

$\Rightarrow \quad 16x^2 - 8x + 1 = 0$

$\Rightarrow \quad 16x^2 - 4x - 4x + 1 = 0$

$\Rightarrow \quad 4x(4x - 1) - 1(4x - 1) = 0$

$\Rightarrow \quad (4x - 1)(4x - 1) = 0$

$\Rightarrow \quad x = \frac{1}{4} \text{ and } x = \frac{1}{4}$

Thus, the required roots are $x = \frac{1}{4}$ and $x = \frac{1}{4}$.

(v) We have, $100x^2 - 20x + 1 = 0$

$\Rightarrow \quad 100x^2 - 10x - 10x + 1 = 0$

$\Rightarrow \quad 10x(10x - 1) - 1(10x - 1) = 0$

$\Rightarrow \quad (10x - 1)(10x - 1) = 0$

$\Rightarrow \quad (10x - 1) = 0 \text{ and } (10x - 1) = 0$

$\Rightarrow \quad x = \frac{1}{10} \text{ and } x = \frac{1}{10}$

Thus, the required roots are $x = \frac{1}{10}$ and $x = \frac{1}{10}$.

NS. 2

Solve the problems :

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs. 750 We would like to find out the number of toys produced on that day.

Ans. **(i)** Let John had x marbles and the Jivanti had $(45 - x)$. When both of them lost 5 marbles then equation becomes, $(x - 5) \times (45 - x - 5) = 124$

$\Rightarrow \quad (x - 5)(40 - x) = 124$

$\Rightarrow \quad x^2 - 45x + 324 = 0$

$\Rightarrow \quad x^2 - 9x - 36x + 324 = 0$

$\Rightarrow \quad x(x - 9) - 36(x - 9) = 0$

$\Rightarrow \quad (x - 9)(x - 36) = 0$

Either $x - 9 = 0 \quad \Rightarrow x = 9$

or $x - 36 = 0 \quad \Rightarrow x = 36$

Thus, $x = 9$ and $x = 36$

$\therefore$ John has 9 marbles and Jivanti had $45 - 9 = 36$ marbles and vice versa.

(ii) Let the number of toys produced on that day be x.

Then cost of 1 toy $= \frac{750}{x}$

$\Rightarrow \quad \frac{750}{x} = 55 - x$

$\Rightarrow \quad x^2 - 55x = -750$

$\Rightarrow \quad x^2 - 55x = + 750 = 0$

$\Rightarrow \quad x^2 - 30x - 25x + 750 = 0$

$\Rightarrow \quad x(x - 30) - 25(x - 30) = 0$

$\Rightarrow (x-30)(x-25)=0$

Either $x-30=0$

$\Rightarrow x=30$ or $x-25=0 \quad \Rightarrow \quad x=25$

Thus, $x=30$ and $x=25$.

NS. 3

Find two numbers whose sum is 27 and product is 182.

Ans. Here, sum of the numbers is 27.

Let one of the numbers be x.

$\therefore$ Other number $=27-x$

According to the condition, Product of the numbers $=182$

$\Rightarrow x(27-x)=182$

$\Rightarrow 27x-x^2=182$

$\Rightarrow x^2-27x+182=0$

$\Rightarrow x^2-13x-14x+182=0$

$\Rightarrow x(x-13)-14(x-13)=0$

$\Rightarrow (x-13)(x-14)=0$

Either $x-13=0$

$\Rightarrow x=13$ or $x-14=0$

$\Rightarrow x=14$

Thus, the required numbers are 13 and 14.

NS. 4

Find two consecutive positive integers, sum of whose squares is 365.

Ans. Let the two consecutive positive integers be x and $(x+1)$

Since the sum of the square of the numbers $=365$

$\therefore x^2+(x+1)^2=365$

$\Rightarrow x^2+[x^2+2x+1]=365$

$\Rightarrow x^2+x^2+2x+1=365$

$\Rightarrow 2x^2+2x+1-365=0$

$\Rightarrow 2x^2+2x-364=0$

$\Rightarrow x^2+x-182=0$

$\Rightarrow x^2+14x-13x-182=0$

$\Rightarrow x(x+14)-13(x+14)=0$

$\Rightarrow (x+14)(x-13)=0$

Either $x+14=0$

$\Rightarrow x=-14$ or $x-13=0$

$\Rightarrow x=13$

Since x has to be a positive integer

$\Rightarrow x=-14$ is rejected.

$\therefore x=13$

$\Rightarrow x+1=13+1=14$

Thus, the required consecutive positive integers are 13 and 14.

NS. 5

The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Ans. Let the base of the given right triangle be 'x' cm

$\therefore$ Its height $=(x-7)$ cm

$\because$ Hypotenuse $=\sqrt{(\text{Base})^2+(\text{Height})^2}$

[By Pythagoras theorem]

$\therefore 13=\sqrt{x^2+(x-7)^2}$

Squaring both sides, we get $169=x^2+(x-7)^2$

$\Rightarrow 169=x^2+x^2-14x+49$

$\Rightarrow 2x^2-14x+49-169=0$

$\Rightarrow x^2-12x+5x-60=0$

$\Rightarrow x(x-12)+5(x-12)=0$

$\Rightarrow (x-12)(x+5)=0$

Either $x-12=0$

$\Rightarrow x=12$ or $x+5=0$

$\Rightarrow x=-5$

But the sides of a triangle can never be negative

$\Rightarrow x=-5$ is rejected.

$\Rightarrow x=12$

$\therefore$ Length of the base $=12$ cm ,

$\Rightarrow$ Length of the height $=(12-7)$cm $=5$ cm

Thus, the required base $=12$ cm and height $=5$ cm.

NS. 6

A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs. 90, find the number of articles produced and the cost of each article.

Ans. Let the number of articles produced in a day = x

$\therefore$ Cost of production of each article = Rs. (2x + 3)

According to the condition, total cost = 90

$\Rightarrow x \times (2x+3) = 90$

$\Rightarrow 2x^2 + 3x = 90$

$\Rightarrow 2x^2 + 3x - 90 = 0$

$\Rightarrow 2x^2 - 12x + 15x - 90 = 0$

$\Rightarrow 2x(x-6) + 15(x-6) = 0$

$\Rightarrow (x-6)(2x+15) = 0$

Either $x - 6 = 0$

$\Rightarrow x = 6$ or $2x + 15 = 0$

$\Rightarrow x = -\dfrac{15}{2}$

But the number of articles produced can never be negative,

$\therefore$ $x = -\dfrac{15}{2}$ is rejected $\Rightarrow$ $x = 6$

$\therefore$ Cost of each article = Rs. $(2 \times 6 + 3)$ = Rs. 15

Thus, the required number of articles produced is 6 and the cost of each article is Rs. 15.

EXERCISE - 4.3

NS. 1

Find the roots of the following quadratic equations, if they exist, by the method of completing the square:

(i) $2x^2 - 7x + 3 = 0$

(ii) $2x^2 + x - 4 = 0$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

(iv) $2x^2 + x + 4 = 0$

Ans. (i) We have, $2x^2 - 7x + 3 = 0$

Dividing throughout by the co-efficient of x^2, we get

$$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

$$\Rightarrow \left\{x^2 - \frac{7}{2}x + \left(\frac{7}{4}\right)^2\right\} - \left(\frac{7}{4}\right)^2 + \frac{3}{2} = 0$$

$$\Rightarrow \left\{x - \frac{7}{4}\right\}^2 - \frac{49}{16} + \frac{3}{2} = 0$$

$$\Rightarrow \left\{x - \frac{7}{4}\right\}^2 - \frac{49}{16} + \frac{24}{16} = 0$$

$$\Rightarrow \left\{x - \frac{7}{4}\right\}^2 - \frac{25}{16} = 0$$

$$\Rightarrow \left\{x - \frac{7}{4}\right\}^2 - \frac{25}{16} = \left(\frac{5}{4}\right)^2$$

$$\Rightarrow x - \frac{7}{4} = \pm\frac{5}{4}$$

Case I : $x - \frac{7}{4} = \frac{5}{4} \Rightarrow x = \frac{5}{4} + \frac{7}{4} \Rightarrow x = \frac{12}{4} = 3$

Case II : $x - \frac{7}{4} = -\frac{5}{4} \Rightarrow x = \frac{-5}{4} + \frac{7}{4}$

$$\Rightarrow x = \frac{2}{4} = \frac{1}{2}$$

Thus, required roots are x = 3 and $x = \dfrac{1}{2}$

(ii) We have, $2x^2 + x - 4 = 0$

Dividing throughout by 2, $x^2 + \dfrac{x}{2} - 2 = 0$

$$\Rightarrow \left\{x^2 + \frac{x}{2} + \left(\frac{1}{4}\right)^2\right\} - \left(\frac{1}{4}\right)^2 - 2 = 0$$

$$\Rightarrow \left(x+\frac{1}{4}\right)^2-\frac{1}{16}-2=0$$

$$\Rightarrow \left(x+\frac{1}{4}\right)^2-\frac{33}{16}=0$$

$$\Rightarrow \left(x+\frac{1}{4}\right)^2-\left(\frac{\sqrt{33}}{4}\right)^2$$

$$\Rightarrow \left(x+\frac{1}{4}\right)^2=\pm\frac{\sqrt{33}}{4}$$

Case I : $x+\frac{1}{4}=\frac{\sqrt{33}}{4}$ $\Rightarrow x=\frac{\sqrt{33}}{4}-\frac{1}{4}$

$$\Rightarrow x=\frac{\sqrt{33}-1}{4}$$

Case II : $x+\frac{1}{4}=-\frac{\sqrt{33}}{4}$ $\Rightarrow x=\frac{-\sqrt{33}}{4}-\frac{1}{4}$

$$\Rightarrow x=\frac{-\sqrt{33}-1}{4}$$

Thus, the required roots are

$$x=\frac{\sqrt{33}-1}{4} \text{ and } x=\frac{-\sqrt{33}-1}{4}$$

(iii) We have $4x^2+4\sqrt{3}x+3=0$

Dividing throughout by 4,

we have $x^2+\sqrt{3}x+\frac{3}{4}=0$

$$\Rightarrow \left\{x^2+\sqrt{3}x+\left(\frac{\sqrt{3}}{2}\right)^2\right\}-\left(\frac{\sqrt{3}}{2}\right)^2+\frac{3}{4}=0$$

$$\Rightarrow \left[x+\frac{\sqrt{3}}{2}\right]^2-\frac{3}{4}+\frac{3}{4}=0$$

$$\Rightarrow \left[x+\frac{\sqrt{3}}{2}\right]^2=0$$

$$\Rightarrow \left(x+\frac{\sqrt{3}}{2}\right)\left(x+\frac{\sqrt{3}}{2}\right)=0$$

$$\Rightarrow x=-\frac{\sqrt{3}}{2} \text{ and } x=-\frac{\sqrt{3}}{2}$$

(iv) $2x^2+x+4=0$

Dividing throughout by 2, we have $x^2+\frac{x}{2}+2=0$

$$\Rightarrow \left[x+\frac{1}{4}\right]^2-\left(\frac{1}{4}\right)^2+2=0$$

$$\Rightarrow \left[x+\frac{1}{4}\right]^2-\frac{1}{16}+2=0$$

$$\Rightarrow \left[x+\frac{1}{4}\right]^2+\frac{31}{16}=0$$

$$\Rightarrow \left[x+\frac{1}{4}\right]^2=-\frac{31}{16}$$

But the square of a number cannot be negative.

$\therefore$ $\left[x+\frac{1}{4}\right]^2$ cannot give a real value.

$\Rightarrow$ There is no real value of x that satisfy the given equation.

NS. 2

Find the roots of the following quadratic equations, using the quadratic formula.

(i) $2x^2-7x+3=0$

(ii) $2x^2+x-4=0$

(iii) $4x^2+4\sqrt{3}x+3=0$

(iv) $2x^2+x+4=0$

Ans. (i) Comparing the given equation with $ax^2+bx+c=0$, we have $a=2$, $b=-7$, $c=3$

$\therefore$ $b^2-4ac=(-7)^2-4(2)(3)=49-24=25\geq 0$

Since $b^2-4ac>0$

$\therefore$ The given equation has real roots. The roots are given by.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{-(-7) \pm \sqrt{(25)}}{2(2)} = \frac{7 \pm 5}{4}$$

Taking positive sign, $x = \frac{7+5}{4} = \frac{12}{4} = 3$

Taking negative sign, $x = \frac{7-5}{4} = \frac{2}{4} = \frac{1}{2}$

Thus, the roots of the given equation are x = 3 and

$x = \frac{1}{2}$

(ii) Comparing the given equation with

$ax^2 + bx + c = 0$ we have $a = 2, b = 1, c = -4$

$\therefore \quad b^2 - 4ac = (1)^2 - 4(2)(-4) = 1 + 32 = 33 > 0$

Since $b^2 - 4ac > 0$

$\therefore$ The given equation has real roots. The roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{-1 \pm \sqrt{33}}{2(2)} = \frac{-1 \pm \sqrt{33}}{4}$$

Taking positive sign, $x = \frac{-1+\sqrt{33}}{4}$

Taking negative sign, $x = \frac{-1-\sqrt{33}}{4}$

Thus, the required roots are

$x = \frac{-1+\sqrt{33}}{4}$ and $x = \frac{-1-\sqrt{33}}{4}$

(iii) Comparing the given equation with

$ax^2 + bx + c = 0$, we have $a = 4,\ b = 4\sqrt{3}, c = 3$

$\therefore \quad b^2 - 4ac = (4\sqrt{3})^2 - 4(4)(3)$

$= [16 \times 3] - 48 = 48 - 48 = 0$

Since $b^2 - 4ac = 0$

$\therefore$ The given equation has real and equal roots,

which are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\Rightarrow \quad x = \frac{-4\sqrt{3} \pm \sqrt{0}}{2(4)} = \frac{-4\sqrt{3} \pm 0}{8} = \frac{-\sqrt{3} \pm 0}{2}$$

$\therefore \quad x = \frac{-\sqrt{3}}{2}$ and $x = \frac{-\sqrt{3}}{2}$

(iv) Comparing the given equation with

$ax^2 + bx + c = 0$, we have $a = 2, b = 1, c = 4$

$\therefore \quad b^2 - 4ac = (1)^2 - 4(2)(4) = 1 - 32 = -31 < 0$

Since $b^2 - 4ac$ is less than 0, therefore the given equation does not have real roots.

NS. 3

Find the roots of the following equation:

(i) $x - \frac{1}{x} = 3, x \neq 0$

(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$

Ans. (i) We have, $x - \frac{1}{x} = 3$

$\Rightarrow \quad x^2 - 1 = 3x$

$\Rightarrow \quad x^2 - 3x - 1 = 0$

Comparing (1) with $ax^2 + bx + c = 0$,

we have $a = 1, b = -3, c = -1$

$\therefore \quad b^2 - 4ac\ (-3)^2 - 4(1)(-1) = 9 + 4 = 13 > 0$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{-(-3) \pm \sqrt{13}}{2(1)} = \frac{3 + \sqrt{13}}{2}$$

Taking positive sign, $x = \frac{3+\sqrt{13}}{2}$

Taking negative sign, $x = \frac{3-\sqrt{13}}{2}$

Thus, the required roots of the given equation are

$$x = \frac{3+\sqrt{13}}{2} \text{ and } x = \frac{3-\sqrt{13}}{2}$$

(ii) We have, $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$

$\Rightarrow$ $(x-7)-(x+4) = \frac{11}{30}(x+4)(x-7)$

$\Rightarrow$ $x-7-x-4 = \frac{11}{30}(x^2-3x-28)$

$\Rightarrow$ $-11 \times 30 = 11(x^2 - 3x - 28)$

$\Rightarrow$ $-30 = x^2 - 3x - 28$

$\Rightarrow$ $x^2 - 3x - 28 + 30 = 0$

$\Rightarrow$ $x^2 - 3x + 2 = 0$

Comparing (1) with $ax^2 + bx + c = 0$,

we have $a = 1, b = -3, c = 2$

$\therefore$ $b^2 - 4ac = (-3)^2 - 4(1)(2) = 9 - 8 = 1 > 0$

$\Rightarrow$ The given equation (1) has real roots, which are given by

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$\Rightarrow$ $x = \frac{-(-3) \pm \sqrt{1}}{2(1)} = \frac{3 \pm 1}{2}$

$\Rightarrow$ Taking positive sign, $x = \frac{3+1}{2} = \frac{4}{2} = 2$

$\Rightarrow$ Taking negative sign, $x = \frac{3-1}{2} = 1$

Thus, the required roots are $x = 2$ and $x = 1$

NS. 4

The sum of the recepricals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Ans. Let the present age of Rehman = x

$\therefore$ 3 years ago Rehman's age = (x – 3) years

5 years later Rehman's age = (x + 5) years

Now, according to the condition,

$$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3} \Rightarrow \frac{(x+5)+(x-3)}{(x-3)(x+5)} = \frac{1}{3}$$

$\Rightarrow$ $3[x + 5 + x - 3] = (x - 3)(x + 5)$

$\Rightarrow$ $3[2x + 2] = x^2 + 2x - 15$

$\Rightarrow$ $6x + 6 = x^2 + 2x - 15$

$\Rightarrow$ $x^2 + 2x - 6x - 15 - 6 = 0$

$\Rightarrow$ $x^2 - 4x - 21 = 0$

Now, comparing (1) with $ax^2 + bx + c = 0$, we have

$a = 1, b = -4, c = -21$

$\therefore$ $b^2 - 4ac = (-4)^2 - 4(1)(-21) = 16 + 84 = 100$

Since, $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\Rightarrow$ $x = \frac{-(-4) \pm 100}{2(1)} = \frac{4 \pm 10}{2}$

Taking positive sign, we have $x = \frac{4+10}{2} = \frac{14}{2} = 7$

Taking negative sign, we have $x = \frac{4-10}{2} = \frac{-6}{2} = -3$

Since age connot be negatie,

$\therefore$ $x \neq -3 \quad \Rightarrow x = 7$

So, the present age of Rehman = 7 years.

NS. 5

In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks would have been 210. Find her marks in the two subjects.

Ans. Let Shefali's marks in Mathematics = x

$\therefore$ Marks in English = (30 – x)

[$\because$ Sum of the marks in English and Mathematics = 30]

Now, according to the condition

$(x + 2) \times [(30 - x) - 3] = 210$

$\Rightarrow$ $(x + 2) \times (30 - x - 3) = 210$

$\Rightarrow$ $(x + 2)(-x + 27) = 210$

$\Rightarrow$ $-x^2 + 25x + 54 = 210$

$\Rightarrow$ $-x^2 + 25x + 54 - 210 = 0$

$\Rightarrow$ $-x^2 + 25x - 156 = 0$

$\Rightarrow$ $x^2 - 25x + 156 = 0$ .....(1)

Now, comparing (1) with $ax^2 + bx + c = 0$, we have

$a = 1, b = -25, c = 156$

$\therefore$ $b^2 - 4ac = (-25)^2 - 4(1)(156) = 625 - 624 = 1$

Since, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow$ $x = \dfrac{-(-25) \pm \sqrt{1}}{2(1)}$

$\Rightarrow$ $x = \dfrac{25 \pm 1}{2}$

Taking positive sign, $x = \dfrac{25+1}{2} = \dfrac{26}{2} = 13$

Taking negative sign, $x = \dfrac{25-1}{2} = \dfrac{24}{2} = 12$

When $x = 13$, then $30 - 13 = 17$

When $x = 12$, then $30 - 12 = 18$

Thus, marks in Mathematics = 13

and marks in English = 17

or marks in Mathematics = 12,

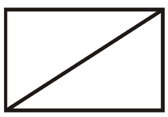
marks in English = 18

NS. 6

The diagonal of a ractangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Ans. Let the shorter side (*i.e.,*, breadth) = x metres

$\therefore$ The longer side (length) = (x + 30) metres



In a rectangle,

$\text{diagonal} = \sqrt{(\text{breadth})^2 + (\text{length})^2}$

$\Rightarrow$ $x + 60 = \sqrt{x^2 + (x+30)^2}$

$\Rightarrow$ $x + 60 = \sqrt{x^2 + x^2 + 60x + 900}$

$\Rightarrow$ $(x + 60)^2 = 2x^2 + 60x + 900$

$\Rightarrow$ $x^2 + 120x + 3600 = 2x^2 + 60x + 900$

$\Rightarrow$ $2x^2 - x^2 + 60x - 120x + 900 - 3600 = 0$

$\Rightarrow$ $x^2 - 60x - 2700 = 0$ .....(1)

Comparing (1) with $ax^2 + bx + c = 0$, we have

$a = 1, b = -60, c = -2700$

$\therefore$ $b^2 - 4ac = (-60)^2 - 4(1)(-2700)$

$\Rightarrow$ $b^2 - 4ac = 3600 + 10800$

$\Rightarrow$ $b^2 - 4ac = 14400$

Since, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow$ $x = \dfrac{-(-60) \pm \sqrt{14400}}{2(1)}$

$\Rightarrow$ $x = \dfrac{60 \pm 120}{2}$

Taking positive sign, $x = \dfrac{60+120}{2} = \dfrac{180}{2} = 90$

Taking negative sign, $x = \dfrac{60-120}{2} = \dfrac{-60}{2} = -30$

Since breadth cannot be negative, so, $x \neq -30$

$\Rightarrow$ $x = 90$

$\therefore$ $x + 30 = 90 + 30 = 120$

Thus, the shorter side = 90 m.

The longer side = 120 m.

NS. 7

The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Ans. Let the larger number be x.

Since, $(\text{smaller number})^2 = 8(\text{larger number})$

$\Rightarrow$ $(\text{smaller number})^2 = 8x$

$\Rightarrow$ smaller number = $\sqrt{8x}$

Now, according to the condition, $x^2 - (\sqrt{8x})^2 = 180$

$\Rightarrow$ $x^2 - 8x = 180$

$\Rightarrow$ $x^2 - 8x - 180 = 0$ ...(1)

Comparing (1) with $ax^2 + bx + c = 0$, we have

$a = 1, b = -8, c = -180$

$\therefore$ $b^2 - 4ac = (-8)^2 - 4(1)(-180) = 64 + 720 = 784$

Since, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore$ $x = \dfrac{-(-8) \pm \sqrt{784}}{2(1)}$

$\Rightarrow$ $x = \dfrac{8 \pm 28}{2}$ $[\because \sqrt{784} = 28]$

Taking positive sign, $x = \dfrac{8+28}{2} = \dfrac{36}{2} = 18$

Taking negative sign, $x = \dfrac{8-28}{2} = \dfrac{-20}{2} = -10$

But $x = -10$ is not admissible,

$\therefore$ The larger number = 18

$\Rightarrow$ Smaller number $\sqrt{8 \times 18} = \sqrt{144} = \pm 12$

Thus, the smaller number = 12 or –12

Thus, the two numbers are 18 and 12, 18 and –12.

NS. 8

A train travels 360 km at a uniform speed. If the speed had been 5 km/hr more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Ans. Let the uniform speed of the train be x km/hr

Since, time taken by the train $= \dfrac{\text{Distance}}{\text{Speed}}$

$\Rightarrow$ time $= \dfrac{360}{x}$ hr

When speed is 5 km/hr more, then time taken is 1 hour less.

$\Rightarrow$ $\dfrac{360}{x+5} - \dfrac{360}{x} = -1$

$\Rightarrow$ $360\left[\dfrac{1}{x+5} - \dfrac{1}{x}\right] = -1$

$\Rightarrow$ $\dfrac{1}{x+5} - \dfrac{1}{x} = \dfrac{-1}{360}$

$\Rightarrow$ $\dfrac{x-(x+5)}{x(x+5)} = \dfrac{-1}{360}$

$\Rightarrow$ $-5 \times 360 = -1(x^2 + 5x)$

$\Rightarrow$ $-5 \times 360 = -x^2 - 5x$

$\Rightarrow$ $x^2 + 5x - 1800 = 0$

Comparing (1) with $ax^2 + bx + c = 0$, we have

$a = 1, b = 5, c = -1800$

$\therefore$ $b^2 - 4ac = (5)^2 - 4(1)(-1800) = 25 + 7200 = 7225$

Since, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore$ $x = \dfrac{-5 \pm \sqrt{7225}}{2(1)}$

$\Rightarrow$ $x = \dfrac{-5 \pm 85}{2(1)}$

Taking positive sign, $x = \dfrac{-5 \pm 85}{2} = \dfrac{80}{2} = 40$

Taking negative sign, $x = \dfrac{-5-85}{2} = \dfrac{-90}{2} = -45$

Since, the speed of a vehicle cannot be negative,

So, $x = -45$ is rejected

Thus, $x = 40$

$\Rightarrow$ speed of the train = 40 km/hr.

NS. 9

Two water taps together can fill a tank in $9\dfrac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately Find the time in which each tap can separately fill the tank.

Ans. Let the smaller tap fills the tank in x hours

$\therefore$ The larger tap fills the tank in (x – 10) hours;

Amount of water flowing through both taps in one hours $= \dfrac{1}{x} + \dfrac{1}{x-10} = \dfrac{x-10+x}{x(x-10)} = \dfrac{2x-10}{x^2-10x}$

Now, according to the condition, $\dfrac{8}{75} = \left(\dfrac{2x-10}{x^2-10x}\right)$

$$\Rightarrow \frac{75(2x-10)}{8(x^2-10x)}=1$$

$$\Rightarrow \frac{150x-750}{8x^2-80x}=1$$

$\Rightarrow$ $8x^2 - 80x = 150x - 750$

$\Rightarrow$ $8x^2 - 80x - 150x + 750 = 0$

$\Rightarrow$ $8x^2 - 230x + 750 = 0$

Comparing (1) with $ax^2 + bx + c = 0$, we get

$a = 8, b = -230, c = 750$

$\therefore$ $b^2 - 4ac = (-230)^2 - 4(8)750$

$= 52900 - 24000 = 28900 > 0$

$$\text{Since, } x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow x=\frac{-(-230)\pm\sqrt{28900}}{2(8)}$$

$$\Rightarrow x=\frac{230\pm170}{16}$$

$$\text{Taking positive sign, } x=\frac{230+170}{16}=\frac{400}{16}=25$$

$$\text{Taking negative sign, } x=\frac{230-170}{16}=\frac{60}{16}=\frac{15}{4}$$

$$\text{For } x=\frac{15}{4},(x-10)=\frac{15}{4}-10=\frac{-25}{4}$$

which is not possible, [$\because$ Time cannot be negative]

$\therefore$ $x = 25$

$\Rightarrow$ $x - 10 = 25 - 10 = 15$

Thus, time to fill the tank by the smaller tap alone is 25 hours and by larger tap alone is 15 hours.

NS. 10

An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11 km/hr more than that of the passenger train, sfind the average speed of the two trains.

Ans. Let the average speed of the passenger train be x km/hr

$\therefore$ Average speed of the express train = (x + 11) km/hr

Total distance covered = 132 km

$$\text{Also, Time}=\frac{\text{Distance}}{\text{Speed}}$$

$$\text{Time taken by the passenger train}=\frac{132}{x}\text{ hours}$$

$$\text{Time taken by the express train}=\frac{132}{x+11}\text{ hours}$$

According to the condition, we get

$$=\frac{132}{x+11}=\left(\frac{132}{x}\right)-1$$

$$\Rightarrow =\frac{132}{x+11}-\frac{132}{x}=-1$$

$$\Rightarrow 132\left[\frac{1}{x+11}-\frac{1}{x}\right]=-1$$

$$\Rightarrow 132\left[\frac{x-x-11}{x(x+11)}\right]=-1$$

$$\Rightarrow 132\left[\frac{-11}{x^2+11x}\right]=-1$$

$\Rightarrow$ $-11(132) = -1(x^2 + 11x)$

$\Rightarrow$ $-1452 = -1(x^2 + 11x)$

$\Rightarrow$ $x^2 + 11x - 1452 = 0$

$$\text{Since, } x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-11\pm\sqrt{5929}}{2(1)}=\frac{-11\pm77}{2}$$

$$\text{Taking positive sign, } x=\frac{-11+77}{2}=\frac{66}{2}=33$$

$$\text{Taking negative sign, } x=\frac{-11-77}{2}=\frac{-88}{2}=-44$$

But average speed cannot be negative so,

$x \neq -44$ $\quad\therefore$ $x = 33$

$\Rightarrow$ Average speed of the passenger train = 33 km/hr

$\therefore$ Average speed of the express train

$= (x + 11)$ km/hr $= (33 + 11)$ km/hr $= 44$ km/hr

NS. 11

Sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the sides of the two squares.

Ans. Let the side of the smaller square be x m

$\Rightarrow$ Perimeter of the smaller square = 4x m

$\therefore$ Perimeter of the larger square = (4x + 24) m

$$\Rightarrow \text{ side of the larger square } = \frac{\text{Perimeter}}{4}$$

$$= \frac{4x+24}{4}\text{m} = \frac{4(x+6)}{4}\text{m} = (x+6)\,\text{m}$$

$\therefore$ Area of the smaller square = $(\text{side})^2 = (x)^2\ m^2$

Area of the larger square = $(x + 6)^2\ m^2$

According to the condition, $x^2 + (x + 6)^2 = 468$

$\Rightarrow x^2 + x^2 + 12x + 36 = 468$

$\Rightarrow 2x^2 + 12x – 432 = 0$

$\Rightarrow x^2 + 6x – 216 = 0$ ...(1)

Comparing (1) with $ax^2 + bx + c = 0$, we get

a = 1, b = 6, c = –216

$\therefore b^2 – 4ac = (6)^2 – 4(1)(–216) = 36 + 864 = 900$

$$\text{Since, } x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\therefore \quad x = \frac{-6 \pm \sqrt{900}}{2(1)} = \frac{-6 \pm 30}{2}$$

Taking positive sign, we have

$$x = \frac{-6+30}{2} = \frac{24}{2} = 12$$

Taking negative sign, we get

$$x = \frac{-6-30}{2} = \frac{-36}{2} = -18$$

But the length of a square cannot be negative

$\therefore$ x = 12

$\Rightarrow$ length of the smaller square = 12 m

Thus, the length of the larger square

= x + 6 = 12 + 6 = 18 m

EXERCISE - 4.4

NS. 1

Find the nature of the roots of the following quadratic equations. If the real roots exist, find them :

(i) $2x^2 – 3x + 5 = 0$

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

(iii) $2x^2 – 6x + 3 = 0$

Ans. (i) Comparing the given quadratic equation with $ax^2 + bx + c = 0$, we have

a = 2, b = –3, c = 5

$\therefore$ Discriminant = $b^2 – 4ac = (–3)^2 – 4(2)(5)$
$= 9 – 40 = –31 < 0$

Since $b^2 – 4ac$ is negative

The given quadratic equation has no real roots.

(ii) Comparing the given quadratic equation with $ax^2 + bx + c = 0$, we get a = 3, $b = -4\sqrt{3}$, c = 4

$\therefore$ $b^2 – 4ac = [-4\sqrt{3}]^2 – 4(3)(4) = (16 \times 3) – 48$
$= 48 – 48 = 0$

Thus, the given quadratic equation has two real roots which are equal. Hence, the roots are

$$\frac{-b}{2a} \text{ and } \frac{-b}{2a} \textit{ i.e., } \frac{-(-4\sqrt{3})}{2\times 3} \text{ and } \frac{-(-4\sqrt{3})}{2\times 3}$$

$$\Rightarrow \frac{4\sqrt{3}}{2\sqrt{3}\times\sqrt{3}} \text{ and } \frac{4\sqrt{3}}{2\sqrt{3}\times\sqrt{3}}$$

$$\Rightarrow \frac{2}{\sqrt{3}} \text{ and } \frac{2}{\sqrt{3}}$$

$$\text{Thus, } x = \frac{2}{\sqrt{3}} \text{ and } x = \frac{2}{\sqrt{3}}$$

(iii) Comparing it with the general quadratic equation, we have a = 2, b = –6, c = 3

$\therefore$ $b^2 – 4ac = (–6)^2 – 4(2)(3) = 36 – 24 = 12 > 0$

The given quadratic equation has two real and distinct roots, which are given by

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{-(-6) \pm \sqrt{12}}{2\times 2} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 \pm \sqrt{3}}{2}$$

Thus, the roots are $x = \frac{3+\sqrt{3}}{2}$ and $x = \frac{3-\sqrt{3}}{2}$

NS. 2

Find the values of k for each of the following quadratic equations so that they have two equal roots

(i) $2x^2 + kx + 3 = 0$ (ii) $kx(x - 2) + 6 = 0$

Ans. (i) Comparing the given quadratic equation with $ax^2 + bx + c = 0$, we get $a = 2$, $b = k$, $c = 3$

$\therefore$ $b^2 - 4ac = (k)^2 - 4(2)(3) = k^2 - 24$

$\because$ For a quadratic equation to have equal roots, $b^2 - 4ac = 0$

$\therefore$ $k^2 - 24 = 0$ $\Rightarrow$ $k = \pm\sqrt{24}$

$\Rightarrow$ $k = \pm 2\sqrt{6}$

Thus, the required values of k are $2\sqrt{6}$ and $-2\sqrt{6}$

(ii) Comparing $kx(x - 2) + 6 = 0$

i.e., $kx^2 - 2kx + 6 = 0$ with $ax^2 + bx + c = 0$, we get $a = k, b = -2k, c = 6$

$\therefore$ $b^2 - 4ac = (-2k)^2 - 4(k)(6) = 4k^2 - 24k$

Since, the roots are real and equal,

$\therefore$ $b^2 - 4ac = 0$

$\Rightarrow$ $4k^2 - 24k = 0$

$\Rightarrow$ $4k(k - 6) = 0$

$\Rightarrow$ $k = 0$ or $k = 6$

But k cannot be 0, otherwise, the given equation is no more quadratic. Thus, the required value of $k = 6$.

NS. 3

Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 sq. metre ? If so, find its length and breadth.

Ans. Let, if possible, the breadth be x metres

$\therefore$ Length = 2x metres

Now, Area = Length × Breadth

$= 2x \times x$ metres^2 $= 2x^2$ sq. metre.

According to the given condition, $2x^2 = 800$

$\Rightarrow$ $x^2 = \frac{800}{2} = 400$

$\Rightarrow$ $x = \pm\sqrt{400} = \pm 20$

Therefore, $x = 20$ and $x = -20$

But $x = -20$ is not possible

[$\because$ breadth cannot be negative]

$\therefore$ $x = 20$ $\Rightarrow$ $2x = 2 \times 20 = 40$

Hence, it is possible to design rectangular mango grove with length = 40 m and breadth = 20 m.

NS. 4

Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Ans. Let the age of one friend = x years

The age of the other friend = (20 – x) years

[$\because$ Sum of their ages is 20 years]

Four years ago

Age of one friend = (x – 4) years Age of other friend = (20 – x – 4) years = (16 – x) years

According to the condition, $(x - 4) \times (16 - x) = 48$

$\Rightarrow$ $16x - 64 - x^2 + 4x = 48$

$\Rightarrow$ $-x^2 + 20x - 64 - 48 = 0$

$\Rightarrow$ $-x^2 + 20x - 112 = 0$

$\Rightarrow$ $x^2 - 20x + 112 = 0$ ...(1)

Here, $a = 1$, $b = -20$ and $c = 112$

$\therefore$ $b^2 - 4ac = (-20)^2 - 4(1)(112)$

$= 400 - 448 = -48 < 0$

Since $b^2 - 4ac$ is less than 0.

The quadratic equation (1) has no real roots.

Thus, the given equation is not possible.

NS. 5

Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2 ? If so, find its length and breadth.

Ans. Let, if possible, the breadth of the rectangle be x m.

Since, the perimeter of the rectangle = 80 m.

$\therefore$ 72 [Length + x] = 80

$\Rightarrow$ $\text{Length} + x = \frac{80}{2} = 40$

$\Rightarrow$ Length = (40 – x) m

$\therefore$ Area = $(40 - x) \times x$ sq.m = $40x - x^2$

Now, according to the given condition,

Area of the rectangle = 400 m^2

$\therefore$ $40x - x^2 = 400$

$\Rightarrow$ $-x^2 + 40x - 400 = 0$

$\Rightarrow$ $x^2 - 40x + 400 = 0$ ...(1)

Comparing (1) with $ax^2 + bx + c = 0$, we get

$a = 1, b = -40, c = 400$

$\therefore$ $b^2 - 4ac = (-40)^2 - 4(1)(400) = 1600 - 1600 = 0$

Thus, the equation (1) has two equal and real roots.

$\because$ $x = \frac{-b}{2a}$ and $x = \frac{-b}{2a}$

$\therefore$ $\text{breadth} = \frac{-(-40)}{2(1)} = \frac{40}{2} = 20$

$\therefore$ Breadth, x = 20 m

$\therefore$ Length = (40 – x) = (40 – 20) m = 20 m.

Since Length = Breadth

$\therefore$ This rectangle is a square.

Space for Notes :

EXERCISE – I

ONLY ONE CORRECT TYPE

1. If one root of quadratic equation $2x^2 + kx - 6 = 0$ is 2 then the other root is.

(A) -1 (B) 2

(C) $-\frac{3}{2}$ (D) $\frac{3}{2}$

2. The roots of $4x^2 - 4ax + (a^2 - b^2) = 0$ are.

(A) $a + b, a - b$ (B) $\frac{a+b}{2}, \frac{a-b}{2}$

(C) $a + b, ab$ (D) $\frac{a+b}{2}, ab$

3. The roots of $x^2 + \left(a + \frac{1}{a}\right) x + 1 = 0$ are $(a \neq 0)$

(A) $a, \frac{1}{a}$ (B) $-a, -\frac{1}{a}$

(C) $a, -\frac{1}{a}$ (D) $-a, \frac{1}{a}$

4. The value of k if $(k + 1)x^2 - 2(k - 1) x + 1 = 0$ has equal roots.

(A) 0, 3 (B) 1, 2

(C) $-3, -2$ (D) 3, 2

5. If one root of the equation $5x^2 + 13x + k = 0$ is reciprocal of the other root then k is

(A) 6 (B) $\frac{1}{6}$

(C) 0 (D) 5

6. If the roots of the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are equal the.

(A) a,b, c are in A.P.

(B) a, c, b are in A.P.

(C) b,a, c in A.P.

(D) none of these

7. If α, β are roots of $x^2 - 7x + 12 = 0$ then the equation whose roots are $3\alpha, 3\beta$ is.

(A) $(3x)^2 - 7(3x) + 12 = 0$

(B) $\left(\frac{x}{3}\right)^2 - 7\left(\frac{x}{3}\right) + 12 = 0$

(C) $(x - 3)^2 - 7(x - 3) + 12 = 0$

(D) $(x + 3)^2 - 7(x + 3) + 12 = 0$

8. If α, β are roots of $2x^2 - 3x - 6 = 0$ then equation whose roots are $\alpha^2 + 2, \beta^2 + 2$ is.

(A) $4x^2 + 49x + 118 = 0$

(B) $4x^2 - 49x + 118 = 0$

(C) $4x^2 - 49x - 118 = 0$

(D) $x^2 - 49x + 118 = 0$

9. Which of the following is quadratic equation ?

(A) $x^2 + 3x + 4 = 0$

(B) $x^3 - 2x^2 + 4 = 0$

(C) $x + \frac{3}{x} = x^2$

(D) $2x^3 - x + 2 = x^2 + 4x - 4$

10. If the product of the roots of $ax^2 + bx + a^2 + 1 = 0$ is -2 then a is

(A) 2 (B) 1

(C) -2 (D) -1

11. If both the roots of the equation $(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$ are real and equal then.

(A) $a + b + c = 0$ (B) $a = b = c$

(C) $a + b = 2c$ (D) $b^2 = ac$

12. The value of $\sqrt{20 + \sqrt{20 + \sqrt{20 + ...\infty}}}$ is.

(A) 4 (B) -5

(C) 5 (D) 20

13. The number of real roots of $2x^4 + 5x^2 + 3 = 0$ is

(A) 3 (B) 2

(C) 4 (D) 0

14. If α,β are roots of the quadratic equation $kx^2 + 4x + 4 = 0$, then the value of k such that $\alpha^2 + \beta^2 = 24$, is.

(A) 1 (B) $-\frac{2}{3}$

(C) –1 (D) none of these

15. The area of a right-angled triangle is 30 m^2. If the base exceeds the altitude by 7 units, then the length of base is.

(A) 12 (B) 13

(C) 6 (D) 5

16. The quadratic equation whose root is $\frac{1}{2+\sqrt{5}}$ is

(A) $x^2 - 4x - 1 = 0$ (B) $x^2 - 4x + 1 = 0$

(C) $x^2 + 4x - 1 = 0$ (D) $x^2 + 4x + 1 = 0$

17. If $ax^2 + bx + c$ is a perfect square then $b^2 =$

(A) 2ac (B) ac

(C) 4ac (D) $\sqrt{2ac}$

18. The value of k so that the quadratic equation, $x^2 - 2x(1 + 3k) + 7(3 + 2k) = 0$ has equal roots is.

(A) 2 (B) 3

(C) 4 (D) 5

19. $ax^2 + ax + 3 = 0$ and $x^2 + x + b = 0$ has one root as 1 then ab =

(A) 3 (B) 3.5

(C) 6 (D) –3

20. If the roots of the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal, then.

(A) $\frac{a}{d} = \frac{b}{c}$ (B) $\frac{a^2+b^2}{c^2} = \frac{b^2+c^2}{d^2}$

(C) $\frac{a}{b} = \frac{c}{d}$ (D) none of these

21. If a and c are such that the quadratic equation $ax^2 - 5x + c = 0$ has 10 as the sum of the roots and also as the products of the roots, then the value of a is.

(1) 5 (2) $\frac{1}{2}$

(3) –5 (4) $-\frac{1}{2}$

22. The value of k such that the sum of the squares of the roots of the quadratic equation $x^2 - 8x + k = 0$ is 40.

(A) 10 (B) 11

(C) 12 (D) –12

23. If –4 is a root of the equation $x^2 + px - 4 = 0$ and the equation $x^2 + px + q = 0$ has equal roots, then the value of q is.

(A) 4 (B) 5

(C) $\frac{4}{9}$ (D) $\frac{9}{4}$

24. If α and β are the roots of $x^2 - 5x + 4 = 0$, the value of $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$ is.

(A) $\frac{4}{27}$ (B) $-\frac{27}{4}$

(C) $\frac{20}{27}$ (D) none of these

25. If α and β are the roots of the equation $3x^2 + 8x + 2 = 0$, then the value of $\alpha^2 + \beta^2$ is.

(A) $\frac{9}{52}$ (B) $-\frac{9}{52}$

(C) $\frac{52}{9}$ (D) none of these

PARAGRAPH TYPE

PASSAGE # I

To represent word problem in the form of quadratic equations, suppose the unknown required quantity can be taken as some variable x (say) and express the given condition in the form of x to form an equation in x. Then we express the equation in the descending powers of x. Thus standard quadratic equation is $ax^2 + bx + c = 0$, $a \neq 0$.

26. The product of two consecutive even integers is 128. Form the quadratic equation to find the integers.

(A) $x^2 + 2x - 128 = 0$

(B) $x^2 + x - 128 = 0$

(C) $x^2 - 2x - 128 = 0$

(D) $x^2 + 2x = -128$

27. A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/hr more, then it would have taken 2 hours less to cover the same distance. The quadratic equation is :

(A) $x^2 + 8x + 480 = 0$

(B) $x^2 + 8x - 1920 = 0$

(C) $x^2 - x + 200 = 0$

(D) $x^2 + 8x - 480 = 0$

28. Sandeep's father is 30 years older than him. The product of their ages 2 years from now will be 400. To find Sandeep's present age, the equation is :

(A) $x^2 + 9x - 13 = 0$

(B) $x^2 + 32x + 400 = 0$

(C) $x^2 + 34x - 336 = 0$

(D) $x^2 - 34x + 90 = 0$

PASSAGE # II

Roots of the quadratic equation of the type $(ax + b)(cx + d) = 0$ are given by the linear equations $ax + b = 0$ and $cx + d = 0$.

29. Find roots of $16x^2 - 9 = 0$.

(A) $x = \frac{4}{3}$ (B) $x = \frac{4}{3}, \frac{-4}{3}$

(C) $x = \frac{2}{3}, \frac{-2}{3}$ (D) $x = \frac{-3}{4}, \frac{3}{4}$

30. Find roots of $a^2x^2 - (a^2b^2 + 1)x + b^2 = 0$.

(A) $x = a, b$ (B) $x = \frac{1}{a^2}, b^2$

(C) $x = a^2b^2, b^2$ (D) $x = \frac{1}{b^2}, a^2$

31. Find roots of the quadratic equation $3x^2 - 2\sqrt{6}x + 2 = 0$.

(A) $x = \frac{9}{4}, \frac{3}{2}$ (B) $x = \frac{4}{3}, \frac{-9}{8}$

(C) $x = \sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}$ (D) $x = \pm\sqrt{\frac{2}{3}}$

MATCH THE COLUMN TYPE

In this section each question has two matching lists. Choices for the correct combination of elements from List - I and List - II are given as options (A), (B), (C) and (D) out of which one is correct.

32. Match the quadratic equations formed in List - I to that in List - II.

List – I	List – II
(P) The product of two consecutive even integers is 528.	(i) $z^2 - 45z + 500 = 0$
(Q) Megha and Latika have 45 chocolates. Both of them lost 3 each, and the product of the chocolates now is 374.	(ii) $x^2 - 15x - 50 = 0$
(R) The hypotenuse of rightangled triangle is 6 more than the shortest side and third side is 3 less than the hypotenuse.	(iii) $n^2 + 2n - 528 = 0$
(S) Difference between two numbers is 5 and the sum of their reciprocals is 1/10.	(iv) $y^2 - 6y - 27 = 0$

(A) (P) → (iii), (Q) → (iv), (R) → (i), (S) → (ii)

(B) (P) → (iii), (Q) → (i), (R) → (iv), (S) → (ii)

(C) (P) → (i), (Q) → (iii), (R) → (ii), (S) → (iv)

(D) (P) → (i), (Q) → (ii), (R) → (iii), (S) → (iv)

33. List - II gives roots of quadratic equations given in List - I match them correctly.

List – I	List – II
(P) $6x^2 + x - 12 = 0$	(i) – 6, 4
(Q) $8x^2 + 16x + 10 = 202$	(ii) 9, 36
(R) $x^2 - 45x + 324 = 0$	(iii) 3, – 1/2
(S) $2x^2 - 5x - 3 = 0$	(iv) – 3/2, 4/3

(A) (P) → (iv), (Q) → (i), (R) → (ii), (S) → (iii)

(B) (P) → (iv), (Q) → (ii), (R) → (i), (S) → (iii)

(C) (P) → (i), (Q) → (ii), (R) → (iii), (S) → (iv)

(D) (P) → (i), (Q) → (iii), (R) → (ii), (S) → (iv)

EXERCISE – II

VERY SHORT ANSWER TYPE

1. What is the nature of roots of the quadratic equation $4x^2 - 12x + 9 = 0$?
2. Write a quadratic equation, sum of whose zeroes is $2\sqrt{3}$ and product is 2.
3. The sum and product of the zeroes of a quadratic polynomial are $-\frac{1}{2}$ and -3 respectively. What is the quadratic equation.
4. What is nature of roots of the quadratic equation $3x^2 - 4\sqrt{3}x + 4 = 0$.
5. Find the value of k so that the following quadratic equation has equal roots $2x^2 - (k-2)x + 1 = 0$.
6. What is the nature of the roots quadratic equation $2x^2 + 5x + 5 = 0$?
7. Find the roots of the quadratic equation $x^2 + 7x + 12 = 0$.
8. For what values of k the quadratic equation $9x^2 - 24x + k = 0$ has equal roots.
9. Find the value of k for which the quadratic equation $x^2 + 5kx + 16 = 0$ has no real roots.
10. Two numbers differ by 3, and their products is 504. Find the numbers.

SHORT ANSWER TYPE

1. Find the quadratic equation one of whose roots is $2 + \sqrt{5}$.
2. For what value of k, $(4-k)x^2 + (2k+4)x + (8k+1)$ is perfect square.
3. Solve by factorization $8x^2 - 22x - 21 = 0$.
4. If one root is equal to the square of the other root of the equation $x^2 + x - k = 0$, what is the value of k ?
5. Find the condition that one root of the equation $ax^2 + bx + c = 0$ may be double of other.

LONG ANSWER TYPE

1. The height of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, form the quadratic equation to find the base of the triangle.
2. Determine the positive values of k for which the equation $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ with both have real roots.
3. Find the number of real roots $3^{2x^2-7x+7} = 9$.
4. Solve the following quadratic equation by factorization method $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$, $a + b \neq 0$.
5. If α, β are the roots of equation $x^2 + px + q = 0$, then find out the quadratic equation whose roots are $1 + \frac{\alpha}{\beta}$, $1 + \frac{\beta}{\alpha}$.

TRUE / FALSE TYPE

1. Every quadratic equation has exactly one root.
2. Every quadratic equation has at least one real root.
3. Every quadratic equation has at least two roots.
4. A quadratic equation has exactly two roots, no more no less.
5. If the coefficient of x^2 and the constant term of a quadratic equation have opposite signs, then the quadratic equation has real roots.

FILL IN THE BLANKS

1. If $b^2 - 4ac = 0$, roots of quadratic equations are real and ....
2. If α and β are zeros of $ax^2 + bx + c = 0$ then $\alpha + \beta = -b$ /............
3. $b^2 - 4ac > 0$, roots of quadratic equations are real and ....
4. A quadratic equations cannot have more then .... roots.
5. For $10x^2 + 10x + 1 = 0$ then $\alpha \times \beta$ ....

ANALYTICAL PROBLEM

1. Find the least positive value of k for which the equation $x^2 + kx + 4 = 0$ has real roots.
2. A rectangular form 60 m long and 40 m wide has two concrete cross roads running in the middle of the park and rest of the park has been used as a lawn. If the area of the lawn is 2109 square m, then what is the width of the road ?
3. If α, β are roots of the equation $x^2 + \sqrt{\alpha}x + \beta = 0$, then $\alpha^2 + \beta^2 =$
4. If $x = 2$ is a root of $3x^2 - 2kx + 2m = 0$ where $m = 3$, then value of 4k is :
5. If the roots of $(b - c)x^2 + (c - a)x + (a - b) = 0$ are equal, then $kb = a + c$. Find k.

NUMERICAL PROBLEMS

1. Non-negative root of $\left(\frac{3x-1}{2x+3}\right)^2 - 5\left(\frac{3x-1}{2x+3}\right) + 4 = 0$ is :
2. If $\alpha = \frac{-b+\sqrt{b^2-12c}}{k}$ and $\beta = \frac{-b-\sqrt{b^2-12c}}{k}$ be two roots of the quadratic equation $3x^2 + bx + c = 0$, then value of 3k is :
3. What is the sum of roots of quadratic equation $4x^2 - 12x + 5 = 0$?
4. The difference of squares of two natural numbers is 45. The square of the smaller number is four times the larger number. The sum of numbers is :
5. The speed of a boat in still water is 8 km/hr. It can go 15 km upstream and 22 km downstream in 5 hours. The speed of the stream is :

Answer Key

EXERCISE–I														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
C	B	B	A	D	A	B	B	A	D	B	C	D	C	A
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
C	C	A	A	C	B	C	D	B	C	A	B	C	D	B
31	32	33												
C	B	A												

EXERCISE II

VERY SHORT ANSWER TYPE

1. Real and equal **2.** $x^2 - 2\sqrt{3}x + 2 = 0$ **3.** $2x^2 + x - 6 = 0$ **4.** Two equal real roots

5. $k = 2 \pm 2\sqrt{2}$ **6.** No real roots **7.** – 4 and – 3 **8.** k = 16 **9.** $\frac{-8}{5} < k < \frac{8}{5}$

10. 21, 24, or –21, –24

SHORT ANSWER TYPE

1. $x^2 - 4x - 1 = 0$ **2.** k = 0, 3 **3.** $x = \frac{7}{2}, \frac{3}{4}$ **4.** k = –1 **5.** $2b^2 = 9ac$

LONG ANSWER TYPE

1. $x^2 - 7x - 60 = 0$ **2.** k = 16 **3.** $\frac{5}{2}, 1$ **4.** x = – a or – b **5.** $qx^2 - p^2x + p^2 = 0$

TRUE / FALSE

1. F **2.** F **3.** F **4.** F **5.** T

FILL IN THE BLANKS

1. Equal **2.** a **3.** Unequal **4.** 2 **5.** 10

ANALYTICAL PROBLEM

1. 4 **2.** 3 **3.** 5 **4.** 18 **5.** 2

NUMERICAL PROBLEMS

1. 4 **2.** 18 **3.** 3 **4.** 15 **5.** 3

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : QUADRATICS EQUATIONS)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
NCERT Exercises			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put “completed” only when you have thoroughly worked through this particular section.

2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

Space for Notes :