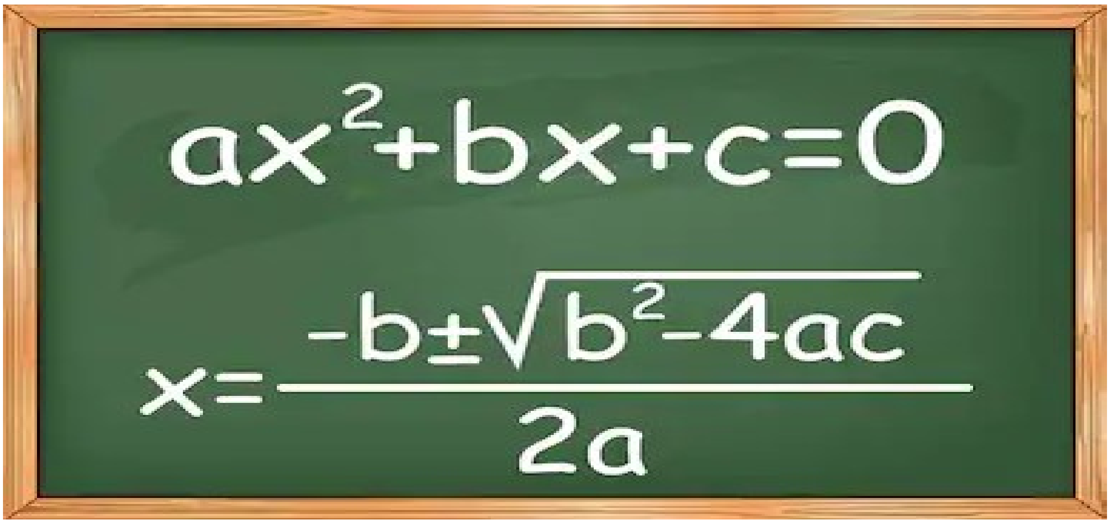
MATRIX

FOUNDATION PROGRAM

AJM MATHEMATICS

QUADRATIC EQUATIONS



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CONTENTS

EXERCISE-1

GENERAL CONCEPT AND QUADRATIC RORMULA

1. $ax^2 + ax + 3 = 0$ and $x^2 + x + b = 0$ has one root as 1 then ab =

(A) 3 (B) 3.5 (C) 6 (D) –3

2. If the product of the roots of $ax^2 + bx + a^2 + 1 = 0$ is –2 then a is :

(A) 2 (B) 1 (C) –2 (D) –1

3. The difference of the roots of $x^2 – 7x – 9 = 0$ is

(A) 7 (B) –9 (C) $\sqrt{85}$ (D) –7

4. If one root of quadratic polynomial $ax^2 – bx + c$ is p then other root is

(A) b – p (B) –b – p (C) $-\frac{b}{a}-p$ (D) $\frac{b}{a}-p$

5. If α, β are roots of equation $x^2 + x – 2 = 0$ then value of $\frac{\alpha\beta^4(\beta+1)^4+\beta\alpha^4(\alpha+1)^4}{\alpha^2+\beta^2+\alpha+\beta}$ is

(A) 2 (B) –2 (C) 4 (D) –4

6. If $\sqrt{2}+3$ is one root of quadratic polynomial having rational coefficients then its sum of roots is equal to

(A) $2\sqrt{2}$ (B) 6 (C) $6+2\sqrt{2}$ (D) 7

7. The value of $x^2 + 2bx + c$ is positive if :

(A) $b^2 – 4ac > 0$ (B) $b^2 < c$ (C) $c^2 < b$ (D) $b^2 – 4ac < 0$

8. If the roots of the equation $x^2 – bx + c = 0$ differ by 2 then which of the following is true?

(A) $c^2 = 4(c + 1)$ (B) $b^2 = 4c + 4$ (C) $c^2 = b + 4$ (D) $b^2 = b(c + 2)$

9. The number of roots of the eqution $x-\frac{2}{x-1}=1-\frac{2}{x-1}$ is :

(A) 0 (B) 1 (C) 2 (D) Infinite

10. The equation $4p^2x^2 + 12pqx + 5q^2 = 0$ has :

(A) Real and equal roots (B) Real and unequal roots

(C) No real roots (D) None of these

11. If roots of equation

$(x – b)(x – c) + (x – a)(x – c) + (x – a) (x – b) = 0$ are always :

(A) Postive (B) Netgative (C) Real (D) None of these

12. If the equation

$(a^2 + b^2)x^2 – 2(ac + bd)x + c^2 + d^2 = 0$ has equal roots than which of the following is true?

(A) ab = cd (B) ad = bc (C) $ad=\frac{2c}{\sqrt{bc}}$ (D) $ab=\sqrt{cd}$

TRANSFORMATION OF ROOTS

13. If α and β are not equal and $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ then equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is :

(A) $3x^2 + 12x + 3 = 0$ (B) $x^2 - 5x - 3 = 0$

(C) $3x^2 - 19x + 3 = 0$ (D) none

14. If α and β are the roots of the equation $ax^2 + bx + c = 0$, then $\frac{1}{a\alpha + b} + \frac{1}{a\beta + b}$ is equal to

(A) $\frac{b}{ac}$ (B) $\frac{-b}{ac}$ (C) $\frac{ac}{b}$ (D) $\frac{-ac}{b}$

15. If the quadratic equations $3x^2 + ax + 1 = 0$ and $2x^2 + bx + 1 = 0$ have a common root, then the value of the expression $5ab - 2a^2 - 3b^2$ is

(A) 0 (B) 1 (C) –1 (D) 2

16. The roots of equation $x^2 + 2|x| + 1 = 0$ has :

(A) One positive and one negative has (B) Both roots positive

(C) Both roots negative (D) No real roots

SPECIAL CASES ON ROOTS

17. If $x^2 + mx + 1$ and $(b - c)x^2 + (c - a)x + (a - b) = 0$ have both roots common then :

(A) $m = -2$ (B) $m = -1$

(C) a, b, c are in A.P. (D) None of these

18. If one root of the equtaion $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ is 1, then other root is ______.

(A) $\frac{b(c-a)}{a(b-c)}$ (B) $\frac{b(b-c)}{c(a-b)}$ (C) $\frac{a(b-c)}{b(c-a)}$ (D) $\frac{c(a-b)}{a(b-c)}$

IDENTITY

19. The value of 'a' for which the equation

$(a^2 + 4a + 3)x^2 + (a^2 - a - 2)x + (a + 1)a = 0$ has more than two roots is given by;

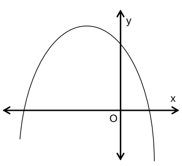
(A) –2 (B) 3 (C) –1 (D) 0

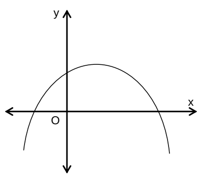
20. The number of values of the pair (a, b) for which $a(x + 1)^2 + b(x^2 - 3x - 2) + x + 1 = 0$ is an identity in x is :

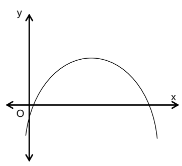
(A) 0 (B) 1 (C) 2 (D) infinite

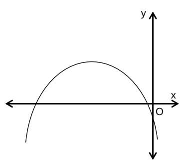
CONSTRUCTION OF GRAPH

21. Which one of the following can best represent the graph of quadratic polynomial $y = ax^2 + bx + c$ (where $a < 0$, $b < 0$ and $c > 0$)?

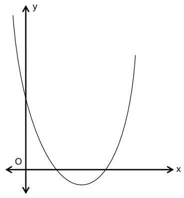
(A) 

(B) 

(C) 

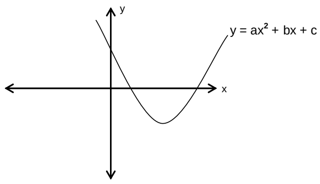
(D) 

22. The graph of $y = ax^2 + bx + c$ is as shown. Which one of the following is not correct?



(A) $a > 0$ (B) $b^2 - 4ac > 0$
(C) $b < 0$ (D) $abc > 0$

23. For the given figure, which of the following is true?



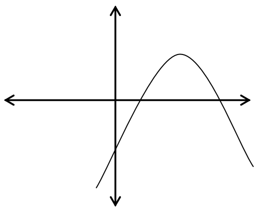
(A) $a > 0, c < 0, b < 0$

(B) $a > 0, c > 0, b < 0$

(C) $a > 0, c < 0, b > 0$

(D) $a < 0, c > 0, b > 0$

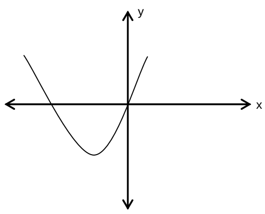
24. If the given figure shows graph of $y = ax^2 + bx + c$ then which of the following is true?



(A) $a > 0$ $b > 0, c < 0$ (B) $a < 0, b > 0, c < 0$

(C) $a < 0, b > 0, c > 0$ (D) $a < 0, b < 0, c < 0$

25. If α and β are zeros of quadratic polynomial $y = ax^2 + bx + c$ then which of the following is true?



(A) $\alpha\beta < 0$ (B) $\alpha\beta > 0$

(C) $\alpha\beta = 0$ (D) $\alpha + \beta > 0$

26. Given that $p(x) = ax^2 + bx + c$ has no real roots and $p(1) < 0$ then which of the following is true?

(A) $c = 0$ (B) $c > 0$

(C) $c < 0$ (D) $c \leq 0$

RANGE

27. Let α and β are the roots of the equation $x^2 - (p-2)x - p - 1 = 0$ ($p \in R$). If $\alpha^2 + \beta^2$ is least, then p equals

(A) 0 (B) 1 (C) 2 (D) 5

28. If x is real, then the maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is

(A) 41 (B) 1 (C) $\frac{1}{4}$ (D) $\frac{17}{7}$

29. If $4x^2 + kx + 3 \geq 0$ for all $x \in R$, then

(A) $k \in \left(-4\sqrt{3}, 4\sqrt{3}\right)$ (B) $k \in \left[-4\sqrt{3}, 4\sqrt{3}\right]$

(C) $k \in \left(-\frac{3}{4}, \frac{3}{4}\right)$ (D) $k \in \left[\frac{-3}{4}, \frac{3}{4}\right]$

30. If a, b are roots of the equation

$x^2 - 2mx + m^2 - 1 = 0$ then the number of integral vlaue of m for which $a, b \in (-2, 4)$ is

(A) 0 (B) 1 (C) 2 (D) 3

LOCATIONS OF ROOTS

31. If the vertex of parabola $y = 2x^2 - 4x + 6$ is (m, n), the parabola whose vertex $\left(2m, \frac{n}{2}\right)$ is

(A) $y = x^2 - 4x + 8$ (B) $y = x^2 + 4x + 6$

(C) $y = -2x^2 - 8x - 6$ (D) $y = -2x^2 + 8x - 8$

32. The number of integral value(s) of 'a' so that the graph of $y = 16x^2 + 8(a+5)x - 7a - 5$ is always above the x-axis is

(A) 12 (B) 13 (C) 14 (D) 15

33. If the graph of $f(x) = x^2 + (3-k)x + k$,

(where $k \in R$) lies above and below x-axis, then k cannot be

(A) –1 (B) 0

(C) 1 (D) 10

34. If the roots of the quadratic polynomial

$f(x) = 2x^2 - 3x + k^2 + 3k - 4$ are of opposite sign, then the set of the value of k is

(A) ϕ (B) $(-4, 1)$ (C) $(4, -1)$ (D) $(-\infty, -4) \cup (1, \infty)$

35. If both roots of $x^2 + bx + c$ are real and given by α and β ($\beta > \alpha$) such that $0 < \alpha < 1$ and $\beta \geq 5$ the which of the following is true?

(A) $(1 + b + c) < 0$ (B) $c(9 + 3b + c) < 0$

(C) $(1 - b + c)(16 - 4b + c) > 0$ (D All of these

EXERCISE-2

SUBJECTIVE /INTERGER TYPE QUESTIONS

1. The value of k such that the equation
   $x^2 + (k - 1)x + 1 = 0$ has such two roots that are equal in magnitude but opposite in signs :
2. If $x^2 - 3x + 2$ is a factor of $x^4 - ax^2 + b = 0$ then the values of a and b are.
3. If both a and b belong to the set {1, 2, 3, 4}, then the number of equations of the form
   $ax^2 + bx + 1 = 0$ having real roots is :
4. If α and β are the roots of the equation
   $x^2 - a(x + 1) - b = 0$, then $(\alpha + 1)(\beta + 1) =$
5. If p and q are the roots of the quadratic equation $x^2 - (a - 2)x - a - 1 = 0$. What is the minimum possible value of $p^2 + q^2$?
6. If α, β are the roots of $x^2 + x + 1 = 0$ and γ, δ are the roots of $x^2 + 3x + 1 = 0$, then
   $(\alpha - \gamma)(\beta + \delta)(\alpha + \delta)(\beta - \gamma) =$
7. Solve $12x^4 - 56x^3 + 89x^2 - 56x + 12 = 0$.
8. Monic quadratic polynomials $P(x)$ and $Q(x)$ have the property that $P(Q(x))$ has zeroes at $x = -23, -21, -17$, and -15 and $Q(P(x))$ has zeros at $x = -59, -57, -51$, and -49. What is the last two digit of the sum of the minimum values of $P(x)$ and $Q(x)$?
9. Given that the real number *s, t* satisfy
   $19s^2 + 99s + 1 = 0$, $t^2 + 99t + 19 = 0$, and $st \neq 1$. Find the value of $\frac{st + 4s + 1}{t}$.
10. Let p be an integer such that both roots of the equation $5x^2 - 5px + (66p - 1) = 0$ are positive integers, Find the value of p.
11. If $x + \sqrt{x^2 - 1} + \frac{1}{x - \sqrt{x^2 - 1}} = 20$ then if $x^2 + \sqrt{x^4 - 1} + \frac{1}{x^2 - \sqrt{x^4 - 1}} = k$, find [$k$].
12. Let α and β be the roots of $x^2 - 5x + 3 = 0$ with
    $\alpha > \beta$. Iϕ $a_n = \alpha^n - \beta^n$ for $n \geq 1$ then the value of $\frac{3a_6 + a_8}{a_7}$ is.

COMPREHENSION TYPE QUESTIONS

Paragraph for Question nos. 13 to 14.

Let us consider the quadratic equation

$(1 + m)x^2 - 2(1 + 3m)x + (1 + 8m) = 0$, where

$m \in R - \{-1\}$.

13. The number of integral values of m such that given quadratic equation has imaginary roots are :

(A) 0 (B) 1

(C) 2 (D) 3

14. The set of values of m such that the given quadratic equation has at least one root negative, is :

(A) $m \in (-\infty, -1)$ (B) $m \in \left(-\frac{1}{8}, \infty\right)$

(C) $m \in \left(-1, -\frac{1}{8}\right)$ (D) $m \in R$

MORE THAN ONE CORRECT TYPE QUESTIONS

15. The quadratic equation $x^2 - 2x - \lambda = 0, \lambda \neq 0$

(A) Can not have a real root if $\lambda < -1$

(B) Can not have a rational root if λ is a perfect square of an integer.

(C) Can not have an integral root if $n^2 - 1 < \lambda < n^2 + 2n, n \in N$

(D) If $\lambda > 3$, then equation has 2 distinct real roots one root less than -1 and oher root greater than 3

16. If one root of the quadratic equation $px^2 + qx + r = 0$ $(p \neq 0)$ is a surd of the form $\frac{\sqrt{a}}{\sqrt{a}+\sqrt{a-b}}$ where p, q, r $\in$ Q and a, b are positive integers which are not perfect square of integer, then the other root is

(A) $\frac{\sqrt{a}}{\sqrt{a}-\sqrt{a-b}}$ (B) $\frac{\sqrt{a}-\sqrt{a-b}}{\sqrt{b}}$

(C) $a + \frac{\sqrt{a(a-b)}}{b}$ (D) $\frac{a+\sqrt{a(a-b)}}{b}$

17. If both the roots of the equation, $(3a + 1)x^2 - (2a + 3b)x + 3 = 0$ tend to infinite then :

(A) $b = 0$ (B) $a = 0$; $b = \infty$

(C) $a = -\frac{1}{3}$ (D) $b = \frac{2}{9}$

18. If the equation $ax^2 + bx + c = 0$; a, b, c $\in$ R and $a \neq 0$ has no real roots then which of the following is/are always correct ?

(A) $(a + b + c)(a - b + c) > 0$

(B) $(a + b + c)(a - 2b + 4c) > 0$

(C) $(a - b + c)(4a - 2b + c) > 0$

(D) $a(b^2 - 4ac) > 0$

MATCH THE COLUMN TYPE QUESTIONS

19. Match the following for range of column - I in column - II

Column - I	Column - II
(A) $f(x)=\frac{x^2+x+2}{x^2+x+1}$	(P) $\left(1,\frac{11}{7}\right]$
(B) $f(x)=\frac{x^2+4x+6}{x^2+4x+5}$	(Q) $\left(1,\frac{7}{3}\right]$
(C) $f(x)=\frac{x^2+3x+5}{x^2+3x+4}$	(R) (1, 3]
(D) $f(x)=\frac{x^2-2x+4}{x^2-2x+2}$	(S) (1, 2]
	(T) (0, 1)

20. Match the following for the quadratic equation $x^2-(k-3)x+k=0$

Column - I	Column - II
(A) Both roots are positive	(P) $(-\infty,1)$
(B) Both roots are negative	(Q) $(9,\infty)$
(C) Both roots are real	(R) (0, 1)
(D) one root <-1, other root >1	(S) (–1, 0)

EXERCISE-3

1. The number of distinct pairs (x, y) of the real numbers satisfying $x = x^3 + y^4$ and $y = 2xy$ is ; **[KVPY 2007]**

(A) 5 (B) 12

(C) 3 (D) 7

2. We want to find a polynomial f(x) of degree n such that $f(1) = \sqrt{2}$ and $f(3) = \pi$. Which of the following is true? **[KVPY 2007]**

(A) There does not exist such a polynomial

(B) There is exactly one such polynomial and it has degree 1

(C) There are infinitely many such polynomials for each $n \geq 1$

(D) There are infinitely many such polynomials for each $n \geq 2$ but not infinitely many for $n = 1$

3. A polynomial p(x) when divided by $x^2 - 3x + 2$ leaves remainder $2x - 3$. Then : **[KVPY 2007]**

(A) p(x) must have a root between 1 and 2

(B) p(x) cannot have a root between 0 and 3

(C) p(x) must have a real root but may or may not be between 0 and 3

(D) p(x) need not have a real root

4. The smallest value the expression $x^2 + 6x + 8$ attains on the set $\{x \in R \mid x^2 - 2x - 8 \leq 0\}$ is : **[KVPY 2008]**

(A) 0 (B) –1

(C) 8 (D) 3

5. The number of pairs of reals (x, y) such that $x = x^2 + y^2$ and $y = 2xy$ is : **[KVPY 2009]**

(A) 4 (B) 3

(C) 2 (D) 1

6. Let $P(x) = 1 + x + x^2 + x^3 + x^4 + x^5$. What is the remainder when $P(x^{12})$ is divided by P(x) ? **[KVPY 2009]**

(A) 0 (B) 6

(C) $1 + x$ (D) $1 + x + x^2 + x^3 + x^4$

7. If $x^2 - 5x + 1 = 0$, then $\frac{x^{10}+1}{x^5}$ has the value : **(NSEJS 2010)**

(A) 2524 (B) 2525 (C) 2424 (D) 2010

8. The graph of the equation $y = 2x^2 + 4x + 3$ has its lowest point at : **(NSEJS 2010)**

(A) (–1, 9) (B) (1, 9)

(C) (–1, 1) (D) (0, 3)

9. If one of the roots of the equation $x^2 - px + q = 0$ is m times the other root then $m/(1 + m^2)$ is equal to : **(NSEJS 2012)**

(A) $\frac{q}{p^2 - 2q}$ (B) $\frac{q}{q^2 - 2p}$ (C) $\frac{p}{q^2 - 2p}$ (D) $\frac{p}{p^2 - 2q}$

10. If a and b are two positive real number such that $\frac{a^2+b^2}{ab}=6$, then a positive value of $\frac{a}{b}$ lies between:

(NSEJS 2013)

(A) 2 and 3 (B) 3 and 4

(C) 4 and 5 (D) 5 and 6

11. It is given that the equation $x^2+ax+20=0$ has integer roots. What is the sum of all possible values of a?

(PRMO 2013)

12. Let $f(x) = x^3 - 3x + b$ and $g(x) = x^2 + bx - 3$, where b is real number. What is the sum of all possible values of b for which the equations $f(x) = 0$ and $g(x) = 0$ have a common root ? **(PRMO 2013)**

13. Natural numbers k, l, p and q are such that if a and b are roots of $x^2 - kx + l = 0$, then $a+\frac{1}{b}$ and $b+\frac{1}{a}$ are the roots of $x^2 - px + q = 0$. What is the sum of possible values of q? **(PRMO 2014)**

14. For a natural number b, let $N(b)$ denote the number of natural numbers a for which the equation $x^2 + ax + b = 0$ has integer roots. What is the smallest value of b for which $N(b) = 6$? **(PRE-RMO 2014)**

15. The equations $x^2 - 4x + k = 0$ and $x^2 + kx - 4 = 0$, where k is a real number, have exactly one common root. What is the value of k ? **(PRMO 2015)**

16. Let α and β be the roots of the equation $x^2 + x - 3 = 0$. Find the value of the expression $4\beta^2 - \alpha^3$.

(PRMO 2016)

17. If the roots of the equation $\frac{x^2-bx}{ax-c}=\frac{m-1}{m+1}$ are equal and of opposite signs, then the value of m is _____.

(NSEJS 2017)

(A) $\frac{ab}{a+b}$ (B) $\frac{a+b}{ab}$

(C) $\frac{a-b}{a+b}$ (D) $\frac{a+b}{a-b}$

18. Let a, b be integers such that all the roots of the equation $(x^2 + ax + 20)(x^2 + 17x + b) = 0$ are negative integers. What is the smallest possible value of $a + b$? **(PRMO 2017)**

19. Suppose a, b are integers and $a + b$ is a root of $x^2 + ax + b = 0$. What is the maximum possible value of b^2?

(PRMO 2018)

20. a, b real numbers. The least value of $a^2 + ab + b^2 - a - 2b$ is : **(NMTC 2018)**

(A) 1 (B) 0

(C) –1 (D) 2

21. Let α and β be the roots of $x^2 - 5x + 3 = 0$ with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$ then the value of $\frac{3a_6 + a_8}{a_7}$ is :

(NSEJS 2019)

(A) 2
(B) 3
(C) 4
(D) 5

22. If the equation $(\alpha^2 - 5\alpha + 6)x^2 + (\alpha^2 - 3\alpha + 2)x + (\alpha^2 - 4) = 0$ has more than two roots, then the value of α is :

(NSEJS 2019)

(A) 2
(B) 3
(C) 1
(D) None of these

23. The sum of the roots of $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$ is zero. The product of roots is : **(NSEJS 2019)**

(A) 0
(B) $\frac{a+b}{2}$
(C) $-\frac{1}{2}(a^2 + b^2)$
(D) $2(a^2 + b^2)$

24. The number of different integers x that satisfy the equation $\left(x^2 - 5x + 5\right)^{\left(x^2 + 4x - 60\right)} = 1$ is : **(NMTC 2019)**

(A) 6
(B) 4
(C) 5
(D) 3

25. Let $p(x) = x^2 + ax + b$ have two distinct real roots. where *a*, *b*, are number. Define $g(x) = p(x^3)$ for all real numbers *x*. Then which of the following

statements are true ?

I. *g* has exactly two distinct real roots a_2b.

II. *g* can have more than two distinct real roots.

III. There exists a real number α such that $g(x) \geq \alpha$ for all real *x*. **[KVPY- 2019]**

(A) Only I
(B) Only I and III
(C) Only II
(D) Only II and III

26. If $x = \sqrt{2} + \sqrt{3} + \sqrt{6}$ is a root $x^4 + ax^3 + bx^3 + bx^2 + cx + d = 0$, where *a, b, c, d* are integers, what is the value of $|a+b+c+d|$? **[PRMO- 2019]**

27. Let $X = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$ and $S = \{(a,b) \in X \times X : x^2 + ax + b$ and $x^3 + bx + a$ have at least a common real zero$\}$. How many elements are there in *S*? **[PRMO- 2021]**

EXERCISE-4

1. If m is chosen in the quadratic equation $\left(m^2+1\right)x^2-3x+\left(m^2+1\right)^2=0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is : **(JEE MAIN APRIL 2020)**

(A) $10\sqrt{5}$ (B) $8\sqrt{3}$

(C) $8\sqrt{5}$ (D) $4\sqrt{3}$

2. Consider the quadratic equation $(c-5)x^2-2cx+(c-4)=0$, $c\neq 5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval (0, 2) and its other root lies in the interval (2, 3). Then the number of elements in S is : **(JEE MAIN APRIL 2020)**

(A) 18 (B) 12

(C) 10 (D) 11

3. Let α and β be two roots of the equation $x^2+2x+2=0$, then $\alpha^{15}+\beta^{15}$ is equal to :

(JEE MAIN APRIL 2020)

(A) –256 (B) 512

(C) –512 (D) 256

4. The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2-11x+\alpha=0$ are rational numbers is : **(JEE MAIN APRIL 2020)**

(A) 3 (B) 2

(C) 4 (D) 5

5. Let α and β be the roots of the equation $x^2-x-1=0$. If $p_k=(\alpha)^k+(\beta)^k$, $k\geq 1$, then which one of the following statements is not true ? **(JEE MAIN JAN. 2020)**

(A) $p_5=p_2\cdot p_3$ (B) $(p_1+p_2+p_3+p_4+p_5)=26$

(C) $p_3=p_5-p_4$ (D) $p_5=11$

6. The number of real roots of the equation, $e^{4x}+e^{3x}-4e^{2x}+e^x+1=0$ is : **(JEE MAIN JAN. 2020)**

(A) 3 (B) 1

(C) 2 (D) 4

7. Let $a, b\in\mathbf{R}, a\neq 0$ be such that the equation, $ax^2-2bx+5=0$ has a repeated root α, which is also a root of the equation, $x^2-2bx-10=0$. If β is the other root of this equation, then $\alpha^2+\beta^2$ is equal to :

(JEE MAIN JAN. 2020)

(A) 28 (B) 25

(C) 26 (D) 24

8. The least positive value of 'a' for which the equation, $2x^2 + (a - 10)x + \frac{33}{2} = 2a$ has real roots is______.

(JEE MAIN JAN. 2020)

9. Let α and β be the roots of the equation, $5x^2 + 6x - 2 = 0$. If $S_n = \alpha^n + \beta^n, n = 1, 2, 3, ...$, then :

(JEE MAIN SEP. 2020)

(A) $6S_6 + 5S_6 = 2S_4$ (B) $5S_6 + 6S_5 = 2S_4$

(C) $5S_6 + 6S_5 + 2S_4 = 0$ (D) $6S_6 + 5S_5 + 2S_4 = 0$

10. If α and β are the roots of equation $x^2 + px + 2 = 0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $2x^2 + 2qx + 1 = 0$, then $\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$ is equal to : **(JEE MAIN SEP. 2020)**

(A) $\frac{9}{4}(9 - q^2)$ (B) $\frac{9}{4}(9 + p^2)$

(C) $\frac{9}{4}(9 - p^2)$ (D) $\frac{9}{4}(9 + q^2)$

11. The set of all real values of λ for which the quadratic equations, $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval (0, 1) is : **(JEE MAIN SEP. 2020)**

(A) (0, 2) (B) (1, 3)

(C) (–3, –1) (D) (2, 4)

12. Let $\lambda \neq 0$ be in R. If α and β are the roots of the equation, $x^2 - x + 2\lambda = 0$ and α and γ are the roots af the equation, $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to : **(JEE MAIN SEP. 2020)**

(A) 27 (B) 9 (C) 36 (D) 18

13. The product of the roots of the equation $9x^2 - 18|x| + 5 = 0$, is : **(JEE MAIN SEP. 2020)**

(A) $\frac{25}{9}$ (B) $\frac{5}{27}$

(C) $\frac{5}{9}$ (D) $\frac{25}{81}$

14. Let p and q be two positive numbers such that $p + q = 2$ and $p^4 + q^4 = 272$. Then p and q are roots of the equation : **(JEE Main 2021)**

(A) $x^2 - 2x + 8 = 0$ (B) $x^2 - 2x + 136 = 0$

(C) $x^2 - 2x + 16 = 0$ (D) $x^2 - 2x + 2 = 0$

15. The integer 'k', for which the inequality
$x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$ is valid for every x in R, is : **(JEE Main 2021)**
(A) 0 (B) 4
(C) 2 (D) 3

16. The sum of all the real roots of the equation $(e^{2x} - 4)(6e^{2x} - 5e^x + 1) = 0$ is : **(JEE Main 2022)**
(A) $\log_e 3$ (B) $-\log_e 3$
(C) $\log_e 6$ (D) $-\log_e 6$

17. For a natural number n, let $\alpha_n = 19^n - 12^n$. Then, the value of $\frac{31\alpha_9 - \alpha_{10}}{57\alpha_8}$ is ________. **(JEE Main 2022)**

18. Let α, β be the roots of the equation $x^2 - \sqrt{2}x + \sqrt{6} = 0$ and $\frac{1}{\alpha^2} + 1, \frac{1}{\beta^2} + 1$ be the roots of the equation $x^2 + ax + b = 0$. Then the roots of the equation $x^2 - (a + b - 2)x + (a + b + 2) = 0$ are : **(JEE MAIN JULY. 2022)**
(A) non-real complex numbers (B) real and both negative
(C) real and both positive (D) real and exactly one of them is positive

19. If for some p, q, r ∈ R, not all have same sign, one of the roots of the equation $(p^2 + q^2)x^2 - 2q(p + r)x + q^2 + r^2 = 0$ is also a root of the equation $x^2 + 2x - 8 = 0$, then $\frac{q^2 + r^2}{p^2}$ is equal to ________. **(JEE MAIN JULY. 2022)**

20. Let α, β (α > β) be the roots of the quadratic equation $x^2 - x - 4 = 0$. If $P_n = \alpha^n - \beta^n$, $n \in N$, then $\frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}}$ is equal to ________. **(JEE MAIN JULY. 2022)**

Answer Key

EXERCISE - 1

1.	A	2.	D	3.	C	4.	D	5.	D	6.	B	7.	B
8.	B	9.	A	10.	B	11.	C	12.	B	13.	A	14.	A
15.	B	16.	D	17.	A	18.	D	19.	C	20.	A	21.	A
22.	D	23.	B	24.	B	25.	C	26.	C	27.	B	28.	A
29.	B	30.	D	31.	C	32.	A	33.	C	34.	B	35.	D

EXERCISE - 2

1. 1   2. $a=4, b=5$ 3. 7   4. 1 –b   5. 5   6. 8

7. $x=\frac{3}{2},\frac{2}{3},2,\frac{1}{2}$ 8. –100   9. 5   10. 76   11. 101   12. 5

13. C   14. C   15. A,B,C,D   16. A,D   17. C, D

18. A,B,C   19. (A) $\to$ (Q) ; (B) $\to$ (S) ; (C) $\to$ (P) ; (D) $\to$ (R)

20. A $\to$ Q, B $\to$ R, C $\to$ P, Q, D $\to$ P

EXERCISE - 3

1.	A	2.	D	3.	A	4.	A	5.	A	6.	B	7.	B
8.	C	9.	A	10.	A	11.	0	12.	0	13.	4	14.	60
15.	3	16.	19	17.	C	18.	25	19.	81	20.	C	21.	D
22.	A	23.	C	24.	A	25.	B	26	93	27.	24		

EXERCISE - 4

1.	C	2.	D	3.	A	4.	A	5.	A	6.	B	7.	B
8.	8	9.	C	10.	C	11.	B	12.	D	13.	D	14	C
15	D	16.	B	17.	4	18.	B	19.	272	20.	16		